

# Using PETSc Solvers in PyLith

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Rice University

PyLith Virtual 2015  
Cyberspace August 24–25, 2015



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assess solver performance,  
and optimize solvers  
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# Outline

- 1 Controlling the Solver
- 2 Where do I begin?
- 3 How do I improve?
- 4 Can We Do It?
- 5 Nonlinear Systems

# Controlling PETSc

All of PETSc can be controlled by **options**

```
-ksp_type gmres
```

```
-start_in_debugger
```

All objects can have a prefix, `-velocity_pc_type jacobi`

All PETSc options can be given to PyLith

```
--petsc.ksp_type=gmres
```

```
--petsc.start_in_debugger
```

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# Examples

We will illustrate options using

PETSc SNES **ex19**, located at

```
$PETSC_DIR/src/snes/examples/tutorials/ex19.c
```

and

PyLith Example **hex8**, located at

```
$PYLITH_DIR/examples/3d/hex8/
```



# Outline

1 Controlling the Solver

**2 Where do I begin?**

3 How do I improve?

4 Can We Do It?

5 Nonlinear Systems

# What is a Krylov solver?

A Krylov solver builds a small model of a linear operator  $A$ , using a subspace defined by

$$\mathcal{K}(A, r) = \text{span}\{r, Ar, A^2r, A^3r, \dots\}$$

where  $r$  is the initial residual.

The small system is solved directly, and the solution is projected back to the original space.

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# What Should I Know About Krylov Solvers?

- They can handle *low-mode* errors
- They need preconditioners
- They do a lot of inner products

# What is a Preconditioner?

A preconditioner  $M$  changes a linear system,

$$M^{-1}Ax = M^{-1}b$$

so that the effective operator is  $M^{-1}A$ , which is hopefully **better** for Krylov methods.

- Preconditioner should be inexpensive
- Preconditioner should accelerate convergence

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# Always start with LU

Always, always start with LU:

- No iterative tolerance
- (Almost) no condition number dependence
- Check for accidental singularity

In parallel, you need a 3rd party package

- MUMPS (`--download-mumps`)
- SuperLU (`--download-superlu_dist`)



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# What if LU fails?

LU will fail for

- Singular problems
- Saddle-point problems

For saddles use `PC_FIELDSPLIT`

- Separately solves each field
- Decomposition is automatic in PyLith
- Autodetect with `-pc_fieldsplit_detect_saddle_point`
- Exact with full Schur complement solve

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  - Look at what you have
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# Outline

- 
- 
- 3 How do I improve?
  - Look at what you have
  - Back off in steps

# What solver did you use?

**Use `-snes_view` or `-ksp_view` to output a description of the solver:**

```
KSP Object:          (fieldsplit_0_)          1 MPI processes
type: fgmres
  GMRES: restart=100, using Classical (unmodified) Gram-
        Schmidt Orthogonalization with no iterative refinemen
  GMRES: happy breakdown tolerance 1e-30
maximum iterations=1, initial guess is zero
tolerances:  relative=1e-09, absolute=1e-50,
              divergence=10000
right preconditioning
has attached null space
using UNPRECONDITIONED norm type for convergence test
```

# What did the convergence look like?

Use `-snes_monitor` **and** `-ksp_monitor`, **or** `-log_summary`:

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Use `-snes_monitor` **and** `-ksp_monitor`, **or** `-log_summary`:

```
0 SNES Function norm 0.207564
1 SNES Function norm 0.0148968
2 SNES Function norm 0.000113968
3 SNES Function norm 6.9256e-09
4 SNES Function norm < 1.e-11
```



# What did the convergence look like?

Use `-snes_monitor` and `-ksp_monitor`, or `-log_summary`:

```
0 KSP Residual norm 1.61409
  Residual norms for mg_levels_1_ solve.
  0 KSP Residual norm 0.213376
  1 KSP Residual norm 0.0192085
Residual norms for mg_levels_2_ solve.
0 KSP Residual norm 0.223226
1 KSP Residual norm 0.0219992
  Residual norms for mg_levels_1_ solve.
  0 KSP Residual norm 0.0248252
  1 KSP Residual norm 0.0153432
Residual norms for mg_levels_2_ solve.
0 KSP Residual norm 0.0124024
1 KSP Residual norm 0.0018736
1 KSP Residual norm 0.02282
```

# What did the convergence look like?

Use `-snes_monitor` and `-ksp_monitor`, or `-log_summary`:

Event	Count		Time (sec)		Flops		Total
	Max	Ratio	Max	Ratio	Max	Ratio	Mflop/s
KSPSetUp	12	1.0	3.6259e-03	1.0	0.00e+00	0.0	0
KSPSolve	3	1.0	4.8937e-01	1.0	8.93e+05	1.0	2
SNESolve	1	1.0	4.9477e-01	1.0	9.22e+05	1.0	2

# Look at timing

Use `-log_summary`:

Event	Time (sec)		Flops		--- Global ---					Total MF/s
	Max	Ratio	Max	Ratio	%T	%f	%M	%L	%R	
VecMDot	1.8904e-03	1.0	2.49e+04	1.0	0	3	0	0	0	13
MatMult	2.1026e-03	1.0	2.65e+05	1.0	0	29	0	0	0	126
PCApply	4.6001e-01	1.0	7.78e+05	1.0	58	84	0	0	64	2
KSPSetUp	3.6259e-03	1.0	0.00e+00	0.0	0	0	0	0	4	0
KSPSolve	4.8937e-01	1.0	8.93e+05	1.0	61	97	0	0	90	2
SNESolve	4.9477e-01	1.0	9.22e+05	1.0	62	100	0	0	92	2

Use `-log_view ::ascii_info_detail` to get this information as a Python module

# Look at timing

Use `-log_summary:`

Event	Time (sec)		Flops		--- Global ---					Total MF/s
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Use `-log_view ::ascii_info_detail` to get this information as a Python module

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- 3 How do I improve?
  - Look at what you have
  - **Back off in steps**

# Weaken the KSP

GMRES  $\implies$  BiCGStab

- `-ksp_type bcgs`
- Less storage
- Fewer dot products (less work)
- Variants `-ksp_type bcgsl` and `-ksp_type ibcgs`

Complete **Table** of Solvers and Preconditioners

# Weaken the PC

LU  $\implies$  ILU

- `-pc_type ilu`
- **Less storage and work**

In parallel,

- `Hypre -pc_type hypre -pc_hypre_type euclid`
- `Block-Jacobi -pc_type bjacobi -sub_pc_type ilu`
- `Additive Schwarz -pc_type asm -sub_pc_type ilu`

Default for MG smoother is Chebychev/SOR(2)

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Default for MG smoother is Chebychev/SOR(2)

# Algebraic Multigrid (AMG)

- Can solve elliptic problems
  - Laplace, elasticity, Stokes
- Works for unstructured meshes
- `-pc_type gamg`, `-pc_type ml`,  
`-pc_type hypre -pc_hypre_type boomeramg`
- **CRUCIAL** to have an accurate near-null space
  - `MatSetNearNullSpace()`
  - PyLith provides this automatically
- Use `-pc_mg_log` to put timing in its own log stage

# PC\_FieldSplit

- **Separate solves for block operators**
  - Physical insight for subsystems
  - Have optimal PCs for simpler equations
  - Suboptions `fs_fieldsplit_0_*`
- **Flexibly combine subsolves**
  - **Jacobi:** `fs_pc_fieldsplit_type = additive`
  - **Gauss-Siedel:** `fs_pc_fieldsplit_type = multiplicative`
  - **Schur complement:** `fs_pc_fieldsplit_type = schur`

# Stokes example

The common block preconditioners for Stokes require only options:

# The Stokes System

$$\begin{pmatrix} A & B \\ B^T & 0 \end{pmatrix}$$

# Stokes example

The common block preconditioners for Stokes require only options:

```
-pc_type fieldsplit
-pc_field_split_type additive
-fieldsplit_0_pc_type ml
-fieldsplit_0_ksp_type preonly
-fieldsplit_1_pc_type jacobi
-fieldsplit_1_ksp_type preonly
```

$$\text{PC} \begin{pmatrix} \hat{A} & 0 \\ 0 & I \end{pmatrix}$$

Cohouet & Chabard, Some fast 3D finite element solvers for the generalized Stokes problem, 1988.

# Stokes example

The common block preconditioners for Stokes require only options:

```
-pc_type fieldsplit
-pc_field_split_type multiplic
-fieldsplit_0_pc_type hypre
-fieldsplit_0_ksp_type preonly
-fieldsplit_1_pc_type jacobi
-fieldsplit_1_ksp_type preonly
```

$$PC \begin{pmatrix} \hat{A} & B \\ 0 & I \end{pmatrix}$$

Elman, Multigrid and Krylov subspace methods for the discrete Stokes equations, 1994.

# Stokes example

The common block preconditioners for Stokes require only options:

```
-pc_type fieldsplit
-pc_field_split_type schur
-fieldsplit_0_pc_type gamg
-fieldsplit_0_ksp_type preonly
-fieldsplit_1_pc_type none
-fieldsplit_1_ksp_type minres

-pc_fieldsplit_schur_factorization_type diag
```

$$\text{PC} \begin{pmatrix} \hat{A} & 0 \\ 0 & -\hat{S} \end{pmatrix}$$

May and Moresi, Preconditioned iterative methods for Stokes flow problems arising in computational geodynamics, 2008.

Olshanskii, Peters, and Reusken, Uniform preconditioners for a parameter dependent saddle point problem with application to generalized Stokes interface equations, 2006.

# Stokes example

The common block preconditioners for Stokes require only options:

```
-pc_type fieldsplit
-pc_field_split_type schur
-fieldsplit_0_pc_type gamg
-fieldsplit_0_ksp_type preonly
-fieldsplit_1_pc_type none
-fieldsplit_1_ksp_type minres

-pc_fieldsplit_schur_factorization_type lower
```

$$\text{PC} \begin{pmatrix} \hat{A} & 0 \\ B^T & \hat{S} \end{pmatrix}$$

May and Moresi, Preconditioned iterative methods for Stokes flow problems arising in computational geodynamics, 2008.



# Stokes example

The common block preconditioners for Stokes require only options:

```
-pc_type fieldsplit
-pc_field_split_type schur
-fieldsplit_0_pc_type gamg
-fieldsplit_0_ksp_type preonly
-fieldsplit_1_pc_type none
-fieldsplit_1_ksp_type minres

-pc_fieldsplit_schur_factorization_type upper
```

$$\text{PC} \begin{pmatrix} \hat{A} & B \\ 0 & \hat{S} \end{pmatrix}$$

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# Stokes example

The common block preconditioners for Stokes require only options:

```

-pc_type fieldsplit
-pc_field_split_type schur
-fieldsplit_0_pc_type gamg
-fieldsplit_0_ksp_type preonly
-fieldsplit_1_pc_type lsc
-fieldsplit_1_ksp_type minres

-pc_fieldsplit_schur_factorization_type upper

```

$$\text{PC} \begin{pmatrix} \hat{A} & B \\ 0 & \hat{S}_{\text{LSC}} \end{pmatrix}$$

May and Moresi, Preconditioned iterative methods for Stokes flow problems arising in computational geodynamics, 2008.

Kay, Loghin and Wathen, A Preconditioner for the Steady-State N-S Equations, 2002.

Elman, Howle, Shadid, Shuttleworth, and Tuminaro, Block preconditioners based on approximate commutators, 2006.

# Stokes example

The common block preconditioners for Stokes require only options:

```
-pc_type fieldsplit
-pc_field_split_type schur
-pc_fieldsplit_schur_factorization_type full
```

PC

$$\begin{pmatrix} I & 0 \\ B^T A^{-1} & I \end{pmatrix} \begin{pmatrix} \hat{A} & 0 \\ 0 & \hat{S} \end{pmatrix} \begin{pmatrix} I & A^{-1} B \\ 0 & I \end{pmatrix}$$

# Why use FGMRES?

Flexible GMRES (FGMRES) allows a **different preconditioner** at each step:

- Takes twice the memory
- Needed for iterative PCs
- Avoided sometimes with a careful PC choice

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Okay, Computer Boy,  
Can you do this for  
a real PyLith Example?

# Example 3D Hex8 `step10.cfg`

First, we try LU on the whole problem

```
[pylithapp.petsc]  
snes_view = true  
pc_type = lu
```

FAIL

This is due to the saddle point introduced to handle the fault.

# Example 3D Hex8 `step10.cfg`

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# FAIL

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# Example 3D Hex8 `step10.cfg`

Next, we split fields using `PC_FIELDSPLIT`

```
[pylithapp.timedependent.formulation]
matrix_type = aij
split_fields = True

[pylithapp.petsc]
ksp_max_it = 1000
fs_pc_type = fieldsplit
fs_pc_use_amat = true
fs_pc_fieldsplit_type = additive
fs_fieldsplit_displacement_ksp_type = preonly
fs_fieldsplit_displacement_pc_type = lu
fs_fieldsplit_lagrange_multiplier_ksp_type = preonly
fs_fieldsplit_lagrange_multiplier_pc_type = jacobi
```

Converges in 58 iterations because preconditioner is not that strong

# Example 3D Hex8 `step10.cfg`

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fs_fieldsplit_lagrange_multiplier_ksp_type = preonly
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```

Converges in 58 iterations because preconditioner is not that strong

# Example 3D Hex8 `step10.cfg`

## We need to use a full Schur factorization

```
[pylithapp.petsc]
fs_pc_type = fieldsplit
fs_pc_use_amat = true
fs_pc_fieldsplit_type = schur
fs_pc_fieldsplit_schur_factorization_type = full
fs_fieldsplit_displacement_ksp_type = preonly
fs_fieldsplit_displacement_pc_type = lu
fs_fieldsplit_lagrange_multiplier_pc_type = jacobi
fs_fieldsplit_lagrange_multiplier_ksp_type = gmres
fs_fieldsplit_lagrange_multiplier_ksp_rtol = 1.0e-11
```

Works in one iterate! This is good for checking the physics.

# Example 3D Hex8 `step10.cfg`

## We need to use a full Schur factorization

```
[pylithapp.petsc]
fs_pc_type = fieldsplit
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fs_pc_fieldsplit_type = schur
fs_pc_fieldsplit_schur_factorization_type = full
fs_fieldsplit_displacement_ksp_type = preonly
fs_fieldsplit_displacement_pc_type = lu
fs_fieldsplit_lagrange_multiplier_pc_type = jacobi
fs_fieldsplit_lagrange_multiplier_ksp_type = gmres
fs_fieldsplit_lagrange_multiplier_ksp_rtol = 1.0e-11
```

Works in one iterate! This is good for checking the physics.

# Example 3D Hex8 `step10.cfg`

We can add a user defined preconditioner for the Schur complement

```
[pylithapp.timedependent.formulation]
use_custom_constraint_pc = True
[pylithapp.petsc]
fs_pc_fieldsplit_schur_precondition = user
```

# Example 3D Hex8 `step10.cfg`

We can add a user defined preconditioner for the Schur complement

```
[pylithapp.timedependent.formulation]
use_custom_constraint_pc = True
[pylithapp.petsc]
fs_pc_fieldsplit_schur_precondition = user
```

## The initial convergence

```
0 SNES Function norm 1.547533880440e-02
  Linear solve converged due to CONVERGED_RTOL iterations 30
0 KSP Residual norm 1.158385264202e-02
  Linear solve converged due to CONVERGED_RTOL iterations 30
1 KSP Residual norm 2.231623131220e-13
  Linear solve converged due to CONVERGED_RTOL iterations 1
1 SNES Function norm 1.146037096697e-13
```

# Example 3D Hex8 `step10.cfg`

We can add a user defined preconditioner for the Schur complement

```
[pylithapp.timedependent.formulation]
use_custom_constraint_pc = True
[pylithapp.petsc]
fs_pc_fieldsplit_schur_precondition = user
```

improves to

```
0 SNES Function norm 1.547533880440e-02
  Linear solve converged due to CONVERGED_RTOL iterations 24
  0 KSP Residual norm 1.158385264203e-02
  Linear solve converged due to CONVERGED_RTOL iterations 25
  1 KSP Residual norm 5.404403812155e-14
  Linear solve converged due to CONVERGED_RTOL iterations 1
1 SNES Function norm 2.201265688755e-14
```

and gets much better for larger problems.



# Example 3D Hex8 `step10.cfg`

## You can back off the Schur complement tolerance

```
ksp_type = fgmres  
fs_fieldsplit_lagrange_multiplier_ksp_rtol = 1.0e-05
```

### at the cost of more iterates

```
0 SNES Function norm 1.547533880440e-02  
  Linear solve converged due to CONVERGED_RTOL iterations 10  
0 KSP Residual norm 1.158385275006e-02  
  Linear solve converged due to CONVERGED_RTOL iterations 10  
1 KSP Residual norm 1.743099082956e-07  
  Linear solve converged due to CONVERGED_RTOL iterations 15  
2 KSP Residual norm 9.111124472508e-13  
Linear solve converged due to CONVERGED_RTOL iterations 2  
1 SNES Function norm 2.316766461963e-11
```

# Example 3D Hex8 `step10.cfg`

## You can back off the primal LU solver

```
fs_fieldsplit_displacement_ksp_type = preonly
fs_fieldsplit_displacement_pc_type  = gamg
```

## at the cost of more iterates

```
0 SNES Function norm 1.547533880440e-02
  Linear solve converged due to CONVERGED_RTOL iterations 29
0 KSP Residual norm 1.109773332794e-02
  Linear solve converged due to CONVERGED_RTOL iterations 30
1 KSP Residual norm 9.958569998724e-04
  Linear solve converged due to CONVERGED_RTOL iterations 35
2 KSP Residual norm 7.686581573340e-05
  Linear solve converged due to CONVERGED_RTOL iterations 35
...
8 KSP Residual norm 9.265128158958e-12
  Linear solve converged due to CONVERGED_RTOL iterations 35
9 KSP Residual norm 6.167388559253e-13
  Linear solve converged due to CONVERGED_RTOL iterations 9
1 SNES Function norm 1.040835868344e-12
```

# Example 3D Hex8 `step10.cfg`

## You can restore the behavior with a lower tolerance

```
fs_fieldsplit_displacement_ksp_type = gmres
fs_fieldsplit_displacement_ksp_rtol = 5.0e-10
```

but it is quite sensitive to the tolerance.

```
0 SNES Function norm 1.547533880440e-02
  Linear solve converged due to CONVERGED_RTOL iterations 25
0 KSP Residual norm 1.158385264210e-02
  Linear solve converged due to CONVERGED_RTOL iterations 25
1 KSP Residual norm 5.391583176741e-12
  Linear solve converged due to CONVERGED_RTOL iterations 30
2 KSP Residual norm 1.542775878965e-18
  Linear solve converged due to CONVERGED_RTOL iterations 2
1 SNES Function norm 3.600812193033e-14
```

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# Driven Cavity Problem

## SNES ex19.c

```
./ex19 -lidvelocity 100 -grashof 1e2  
-da_grid_x 16 -da_grid_y 16 -da_refine 2  
-snes_monitor_short -snes_converged_reason -snes_view
```

# Driven Cavity Problem

## SNES ex19.c

```
./ex19 -lidvelocity 100 -grashof 1e2  
-da_grid_x 16 -da_grid_y 16 -da_refine 2  
-snes_monitor_short -snes_converged_reason -snes_view
```

```
lid velocity = 100, prandtl # = 1, grashof # = 100
```

```
0 SNES Function norm 768.116
```

```
1 SNES Function norm 658.288
```

```
2 SNES Function norm 529.404
```

```
3 SNES Function norm 377.51
```

```
4 SNES Function norm 304.723
```

```
5 SNES Function norm 2.59998
```

```
6 SNES Function norm 0.00942733
```

```
7 SNES Function norm 5.20667e-08
```

```
Nonlinear solve converged due to CONVERGED_FNORM_RELATIVE iterations 7
```

# Driven Cavity Problem

## SNES ex19.c

```
./ex19 -lidvelocity 100 -grashof 1e4  
-da_grid_x 16 -da_grid_y 16 -da_refine 2  
-snes_monitor_short -snes_converged_reason -snes_view
```

# Driven Cavity Problem

## SNES ex19.c

```
./ex19 -lidvelocity 100 -grashof 1e4  
-da_grid_x 16 -da_grid_y 16 -da_refine 2  
-snes_monitor_short -snes_converged_reason -snes_view
```

```
lid velocity = 100, prandtl # = 1, grashof # = 10000  
0 SNES Function norm 785.404  
1 SNES Function norm 663.055  
2 SNES Function norm 519.583  
3 SNES Function norm 360.87  
4 SNES Function norm 245.893  
5 SNES Function norm 1.8117  
6 SNES Function norm 0.00468828  
7 SNES Function norm 4.417e-08  
Nonlinear solve converged due to CONVERGED_FNORM_RELATIVE iterations 7
```



# Driven Cavity Problem

## SNES ex19.c

```
./ex19 -lidvelocity 100 -grashof 1e5  
-da_grid_x 16 -da_grid_y 16 -da_refine 2  
-snes_monitor_short -snes_converged_reason -snes_view
```

# Driven Cavity Problem

## SNES ex19.c

```
./ex19 -lidvelocity 100 -grashof 1e5  
-da_grid_x 16 -da_grid_y 16 -da_refine 2  
-snes_monitor_short -snes_converged_reason -snes_view
```

```
lid velocity = 100, prandtl # = 1, grashof # = 100000
```

```
0 SNES Function norm 1809.96
```

```
Nonlinear solve did not converge due to DIVERGED_LINEAR_SOLVE iterations C
```

# Driven Cavity Problem

## SNES ex19.c

```
./ex19 -lidvelocity 100 -grashof 1e5  
-da_grid_x 16 -da_grid_y 16 -da_refine 2 -pc_type lu  
-snes_monitor_short -snes_converged_reason -snes_view
```

```
lid velocity = 100, prandtl # = 1, grashof # = 100000  
0 SNES Function norm 1809.96  
1 SNES Function norm 1678.37  
2 SNES Function norm 1643.76  
3 SNES Function norm 1559.34  
4 SNES Function norm 1557.6  
5 SNES Function norm 1510.71  
6 SNES Function norm 1500.47  
7 SNES Function norm 1498.93  
8 SNES Function norm 1498.44  
9 SNES Function norm 1498.27  
10 SNES Function norm 1498.18  
11 SNES Function norm 1498.12  
12 SNES Function norm 1498.11  
13 SNES Function norm 1498.11  
14 SNES Function norm 1498.11  
...
```

# Why isn't SNES converging?

- The Jacobian is wrong (maybe only in parallel)
  - Check with `-snes_check_jacobian`  
`-snes_check_jacobian_view`
- The linear system is not solved accurately enough
  - Check with `-pc_type lu`
  - Check `-ksp_monitor_true_residual`, try right preconditioning
- The Jacobian is singular with inconsistent right side
  - Use **MatNullSpace** to inform the **KSP** of a known null space
  - Use a different Krylov method or preconditioner
- The nonlinearity is just really strong
  - Run with `-info` or `-snes_ls_monitor` to see line search
  - Try using trust region instead of line search `-snes_type tr`
  - Try grid sequencing if possible `-snes_grid_sequence`
  - Use a continuation

# Nonlinear Preconditioning

**PC** preconditions **KSP**

```
-ksp_type gmres
```

```
-pc_type richardson
```

**SNES** preconditions **SNES**

```
-snes_type ngmres
```

```
-npc_snes_type nrichardson
```

# Nonlinear Preconditioning

**PC** preconditions **KSP** **SNES** preconditions **SNES**

```
-ksp_type gmres
```

```
-snes_type ngmres
```

```
-pc_type richardson
```

```
-npc_snes_type nrichardson
```

# Nonlinear Use Cases

## Warm start Newton

```
-snes_type newtonls  
-npc_snes_type nrichardson -npc_snes_max_it 5
```

## Cleanup noisy Jacobian

```
-snes_type ngmres -snes_ngmres_m 5  
-npc_snes_type newtonls
```

## Additive-Schwarz Preconditioned Inexact Newton

```
-snes_type aspin -snes_npc_side left  
-npc_snes_type nasm -npc_snes_nasm_type restrict
```

# Nonlinear Preconditioning

```
./ex19 -lidvelocity 100 -grashof 5e4 -da_refine 4 -snes_monitor_short  
-snes_type newtonls -snes_converged_reason  
-pc_type lu
```

```
lid velocity = 100, prandtl # = 1, grashof # = 50000  
 0 SNES Function norm 1228.95  
 1 SNES Function norm 1132.29  
 2 SNES Function norm 1026.17  
 3 SNES Function norm 925.717  
 4 SNES Function norm 924.778  
 5 SNES Function norm 836.867  
  ⋮  
21 SNES Function norm 585.143  
22 SNES Function norm 585.142  
23 SNES Function norm 585.142  
24 SNES Function norm 585.142  
  ⋮
```



# Nonlinear Preconditioning

```
./ex19 -lidvelocity 100 -grashof 5e4 -da_refine 4 -snes_monitor_short  
-snes_type fas -snes_converged_reason  
-fas_levels_snes_type gs -fas_levels_snes_max_it 6
```

```
lid velocity = 100, prandtl # = 1, grashof # = 50000
```

```
0 SNES Function norm 1228.95
```

```
1 SNES Function norm 574.793
```

```
2 SNES Function norm 513.02
```

```
3 SNES Function norm 216.721
```

```
4 SNES Function norm 85.949
```

```
Nonlinear solve did not converge due to DIVERGED_INNER iterations 4
```

# Nonlinear Preconditioning

```
./ex19 -lidvelocity 100 -grashof 5e4 -da_refine 4 -snes_monitor_short  
-snes_type fas -snes_converged_reason  
-fas_levels_snes_type gs -fas_levels_snes_max_it 6  
-fas_coarse_snes_converged_reason
```

```
lid velocity = 100, prandtl # = 1, grashof # = 50000  
0 SNES Function norm 1228.95  
  Nonlinear solve converged due to CONVERGED_FNORM_RELATIVE its 12  
1 SNES Function norm 574.793  
  Nonlinear solve did not converge due to DIVERGED_MAX_IT its 50  
2 SNES Function norm 513.02  
  Nonlinear solve did not converge due to DIVERGED_MAX_IT its 50  
3 SNES Function norm 216.721  
  Nonlinear solve converged due to CONVERGED_FNORM_RELATIVE its 22  
4 SNES Function norm 85.949  
  Nonlinear solve did not converge due to DIVERGED_LINE_SEARCH its 42  
Nonlinear solve did not converge due to DIVERGED_INNER iterations 4
```

# Nonlinear Preconditioning

```
./ex19 -lidvelocity 100 -grashof 5e4 -da_refine 4 -snes_monitor_short  
-snes_type fas -snes_converged_reason  
-fas_levels_snes_type gs -fas_levels_snes_max_it 6  
-fas_coarse_snes_linesearch_type basic  
-fas_coarse_snes_converged_reason
```

```
lid velocity = 100, prandtl # = 1, grashof # = 50000  
0 SNES Function norm 1228.95  
  Nonlinear solve converged due to CONVERGED_FNORM_RELATIVE its 6  
:  
47 SNES Function norm 78.8401  
  Nonlinear solve converged due to CONVERGED_FNORM_RELATIVE its 5  
48 SNES Function norm 73.1185  
  Nonlinear solve converged due to CONVERGED_FNORM_RELATIVE its 6  
49 SNES Function norm 78.834  
  Nonlinear solve converged due to CONVERGED_FNORM_RELATIVE its 5  
50 SNES Function norm 73.1176  
  Nonlinear solve converged due to CONVERGED_FNORM_RELATIVE its 6  
:  
:
```

# Nonlinear Preconditioning

```
./ex19 -lidvelocity 100 -grashof 5e4 -da_refine 4 -snes_monitor_short
-snes_type nrichardson -npc_snes_max_it 1 -snes_converged_reason
-npc_snes_type fas -npc_fas_coarse_snes_converged_reason
-npc_fas_levels_snes_type gs -npc_fas_levels_snes_max_it 6
-npc_fas_coarse_snes_linesearch_type basic
```

```
lid velocity = 100, prandtl # = 1, grashof # = 50000
 0 SNES Function norm 1228.95
   Nonlinear solve converged due to CONVERGED_FNORM_RELATIVE its 6
 1 SNES Function norm 552.271
   Nonlinear solve converged due to CONVERGED_FNORM_RELATIVE its 27
 2 SNES Function norm 173.45
   Nonlinear solve converged due to CONVERGED_FNORM_RELATIVE its 45
  :
43 SNES Function norm 3.45407e-05
   Nonlinear solve converged due to CONVERGED_SNORM_RELATIVE its 2
44 SNES Function norm 1.6141e-05
   Nonlinear solve converged due to CONVERGED_SNORM_RELATIVE its 2
45 SNES Function norm 9.13386e-06
Nonlinear solve converged due to CONVERGED_FNORM_RELATIVE iterations 45
```

# Nonlinear Preconditioning

```
./ex19 -lidvelocity 100 -grashof 5e4 -da_refine 4 -snes_monitor_short
-snes_type ngmres -npc_snes_max_it 1 -snes_converged_reason
-npc_snes_type fas -npc_fas_coarse_snes_converged_reason
-npc_fas_levels_snes_type gs -npc_fas_levels_snes_max_it 6
-npc_fas_coarse_snes_linesearch_type basic
```

```
lid velocity = 100, prandtl # = 1, grashof # = 50000
 0 SNES Function norm 1228.95
   Nonlinear solve converged due to CONVERGED_FNORM_RELATIVE its 6
 1 SNES Function norm 538.605
   Nonlinear solve converged due to CONVERGED_FNORM_RELATIVE its 13
 2 SNES Function norm 178.005
   Nonlinear solve converged due to CONVERGED_FNORM_RELATIVE its 24
  :
27 SNES Function norm 0.000102487
   Nonlinear solve converged due to CONVERGED_FNORM_RELATIVE its 2
28 SNES Function norm 4.2744e-05
   Nonlinear solve converged due to CONVERGED_SNORM_RELATIVE its 2
29 SNES Function norm 1.01621e-05
   Nonlinear solve converged due to CONVERGED_FNORM_RELATIVE iterations 29
```

# Nonlinear Preconditioning

```

./ex19 -lidvelocity 100 -grashof 5e4 -da_refine 4 -snes_monitor_short
-snes_type ngmres -npc_snes_max_it 1 -snes_converged_reason
-npc_snes_type fas -npc_fas_coarse_snes_converged_reason
-npc_fas_levels_snes_type newtonls -npc_fas_levels_snes_max_it 6
-npc_fas_levels_snes_linesearch_type basic
-npc_fas_levels_snes_max_linear_solve_fail 30
-npc_fas_levels_ksp_max_it 20 -npc_fas_levels_snes_converged_reason
-npc_fas_coarse_snes_linesearch_type basic
lid velocity = 100, prandtl # = 1, grashof # = 50000
0 SNES Function norm 1228.95
  Nonlinear solve did not converge due to DIVERGED_MAX_IT its 6
  :
  Nonlinear solve converged due to CONVERGED_SNORM_RELATIVE its 1
  :
1 SNES Function norm 0.1935
2 SNES Function norm 0.0179938
3 SNES Function norm 0.00223698
4 SNES Function norm 0.000190461
5 SNES Function norm 1.6946e-06
Nonlinear solve converged due to CONVERGED_FNORM_RELATIVE iterations 5

```

# Nonlinear Preconditioning

```
./ex19 -lidvelocity 100 -grashof 5e4 -da_refine 4 -snes_monitor_short  
-snes_type composite -snes_composite_type additiveoptimal  
-snes_composite_sneses fas,newtonls -snes_converged_reason  
-sub_0_fas_levels_snes_type gs -sub_0_fas_levels_snes_max_it 6  
-sub_0_fas_coarse_snes_linesearch_type basic  
-sub_1_snes_linesearch_type basic -sub_1_pc_type mg
```

```
lid velocity = 100, prandtl # = 1, grashof # = 50000
```

```
0 SNES Function norm 1228.95
```

```
1 SNES Function norm 541.462
```

```
2 SNES Function norm 162.92
```

```
3 SNES Function norm 48.8138
```

```
4 SNES Function norm 11.1822
```

```
5 SNES Function norm 0.181469
```

```
6 SNES Function norm 0.00170909
```

```
7 SNES Function norm 3.24991e-08
```

```
Nonlinear solve converged due to CONVERGED_FNORM_RELATIVE iterations 7
```

# Nonlinear Preconditioning

```
./ex19 -lidvelocity 100 -grashof 5e4 -da_refine 4 -snes_monitor_short  
-snes_type composite -snes_composite_type multiplicative  
-snes_composite_sneses fas,newtonls -snes_converged_reason  
-sub_0_fas_levels_snes_type gs -sub_0_fas_levels_snes_max_it 6  
-sub_0_fas_coarse_snes_linesearch_type basic  
-sub_1_snes_linesearch_type basic -sub_1_pc_type mg
```

```
lid velocity = 100, prandtl # = 1, grashof # = 50000
```

```
0 SNES Function norm 1228.95
```

```
1 SNES Function norm 544.404
```

```
2 SNES Function norm 18.2513
```

```
3 SNES Function norm 0.488689
```

```
4 SNES Function norm 0.000108712
```

```
5 SNES Function norm 5.68497e-08
```

```
Nonlinear solve converged due to CONVERGED_FNORM_RELATIVE iterations 5
```



# Nonlinear Preconditioning

Solver	T	N. It	L. It	Func	Jac	PC	NPC
$(\mathcal{N} \setminus \mathcal{K} - \text{MG})$	9.83	17	352	34	85	370	–
NGMRES $_{-R}$ $(\mathcal{N} \setminus \mathcal{K} - \text{MG})$	7.48	10	220	21	50	231	10
FAS	6.23	162	0	2382	377	754	–
FAS + $(\mathcal{N} \setminus \mathcal{K} - \text{MG})$	8.07	10	197	232	90	288	–
FAS * $(\mathcal{N} \setminus \mathcal{K} - \text{MG})$	4.01	5	80	103	45	125	–
NRICH $_{-L}$ FAS	3.20	50	0	1180	192	384	50
NGMRES $_{-R}$ FAS	1.91	24	0	447	83	166	24

# Nonlinear Preconditioning

See discussion in:

## **Composing scalable nonlinear solvers,**

Peter Brune, Matthew Knepley, Barry Smith, and Xuemin Tu,

ANL/MCS-P2010-0112, Argonne National Laboratory, 2012.

<http://www.mcs.anl.gov/uploads/cels/papers/P2010-0112.pdf>

# Other Solver Issues

For any other solver problems,

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