Modeling plasticity throughout the earthquake cycle with ruptures into near surface sediments

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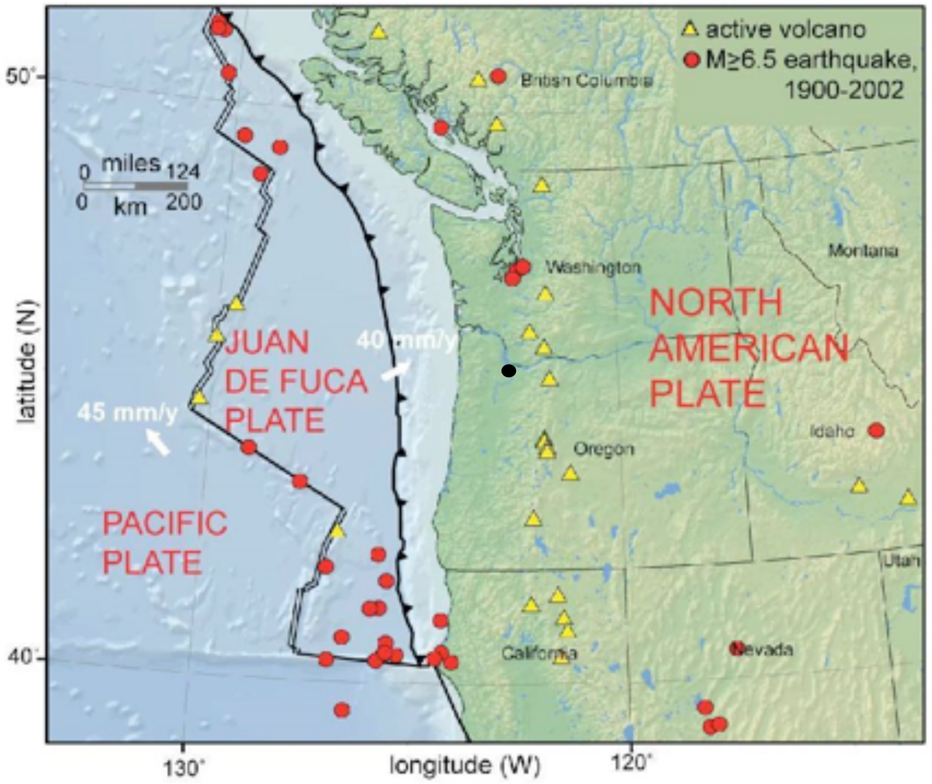


Collaborators: Eric Dunham, Jeremy Kozdon, Steve Day and Arash Khosravifar

MODELING the earthquake cycle: to understand earthquake nucleation and propagation, how and when tectonic deformation is accommodated, to better understand/explain observations.

San Andreas Fault

Modeling challenges

- Multiscale features in space & time.
- Unknown initial conditions.
- Complex fault geometries/ structures.
- Material heterogeneties and inelastic material response.

Recent Advances (incomplete list): Kaneko et al. (2011, coupled, heterog.), Barbot et al. (2012, coupled, integrated), Aagaard et al. (2013, coupled, complex fault geometries), Allison and Dunham (2017, nonlinear viscoelastic), Thompson and Meade.

Our Modeling Framework:

Rate-and-state friction:

$$\tau = \sigma_n f(V, \psi)$$

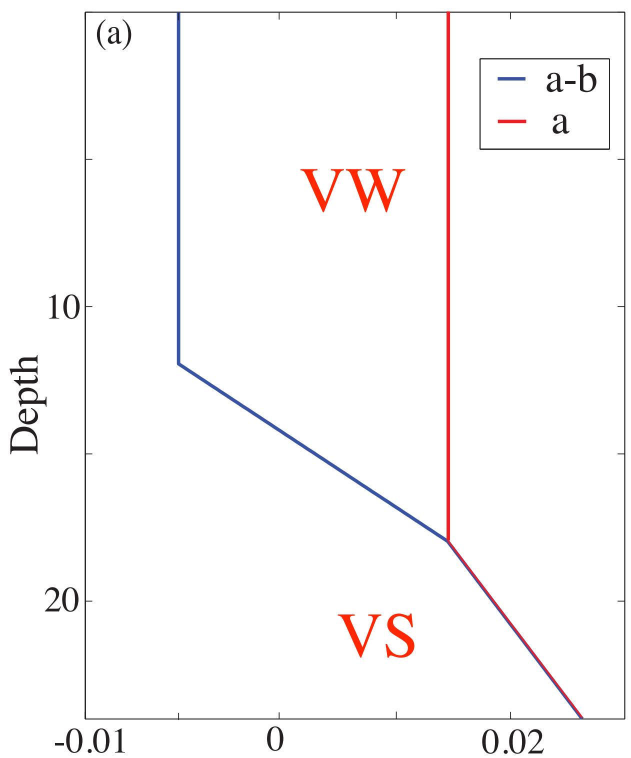
sum of quasi static and radiation damping stress

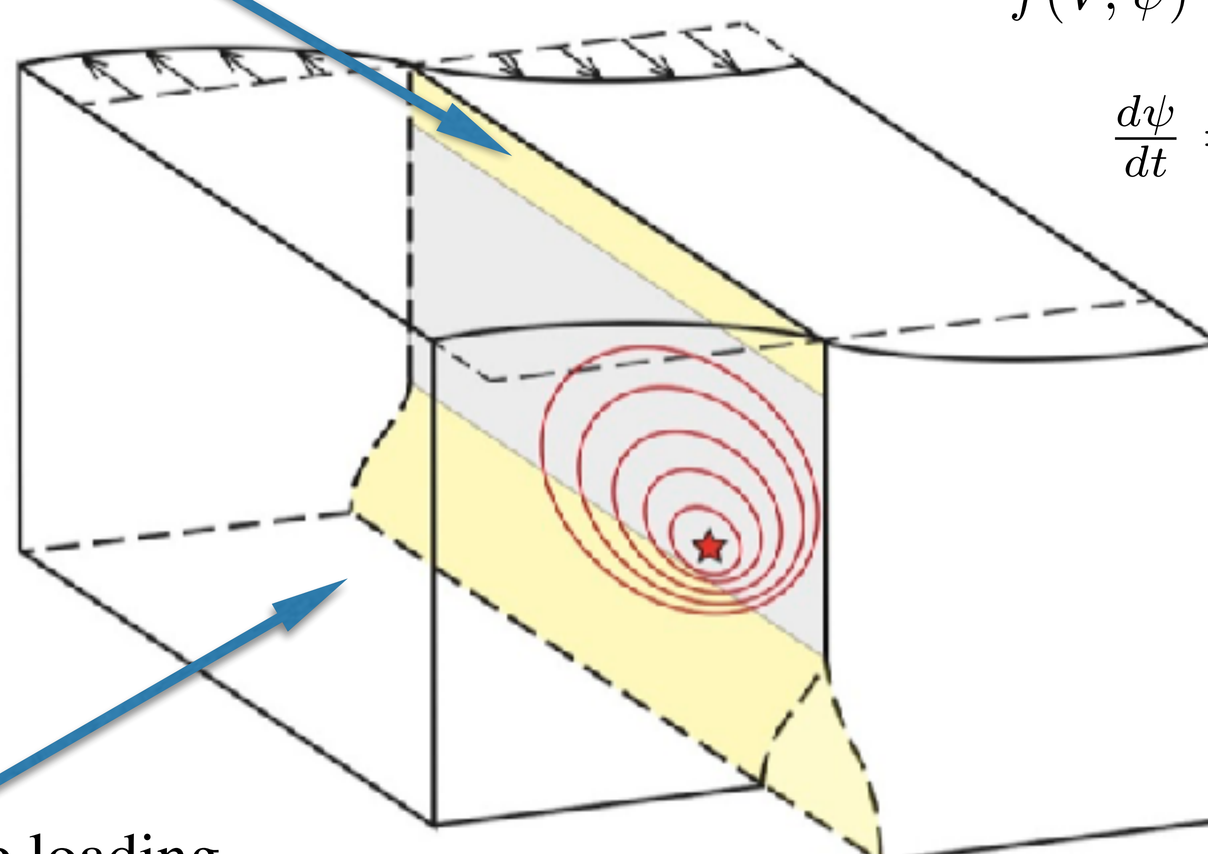
Vertical, strike-slip fault with rate-and-state friction

Finite Difference Method. Off-fault material: heterogeneous, anisotropic, elastoplastic

$$f(V, \psi) = a \sinh^{-1}\left(\frac{V}{2V_0} e^{\psi/a}\right)$$

$$\frac{d\psi}{dt} = \frac{bV_0}{D_c}\left(e^{(f_0-\psi)/b} - \frac{V}{V_0}\right)$$





Remote loading

Events nucleate spontaneously, with inertial effects captured through radiation damping.

2D Antiplane Motion

Governing equations with Hooke's law:

$$0 = \nabla \cdot \sigma \quad \rightarrow \quad 0 = \nabla \cdot (C : \epsilon) \tag{1}$$

$$0 = \frac{\partial}{\partial y}\left[\mu(y,z)\frac{\partial u}{\partial y}\right] + \frac{\partial}{\partial z}\left[\mu(y,z)\frac{\partial u}{\partial z}\right] \tag{2a}$$

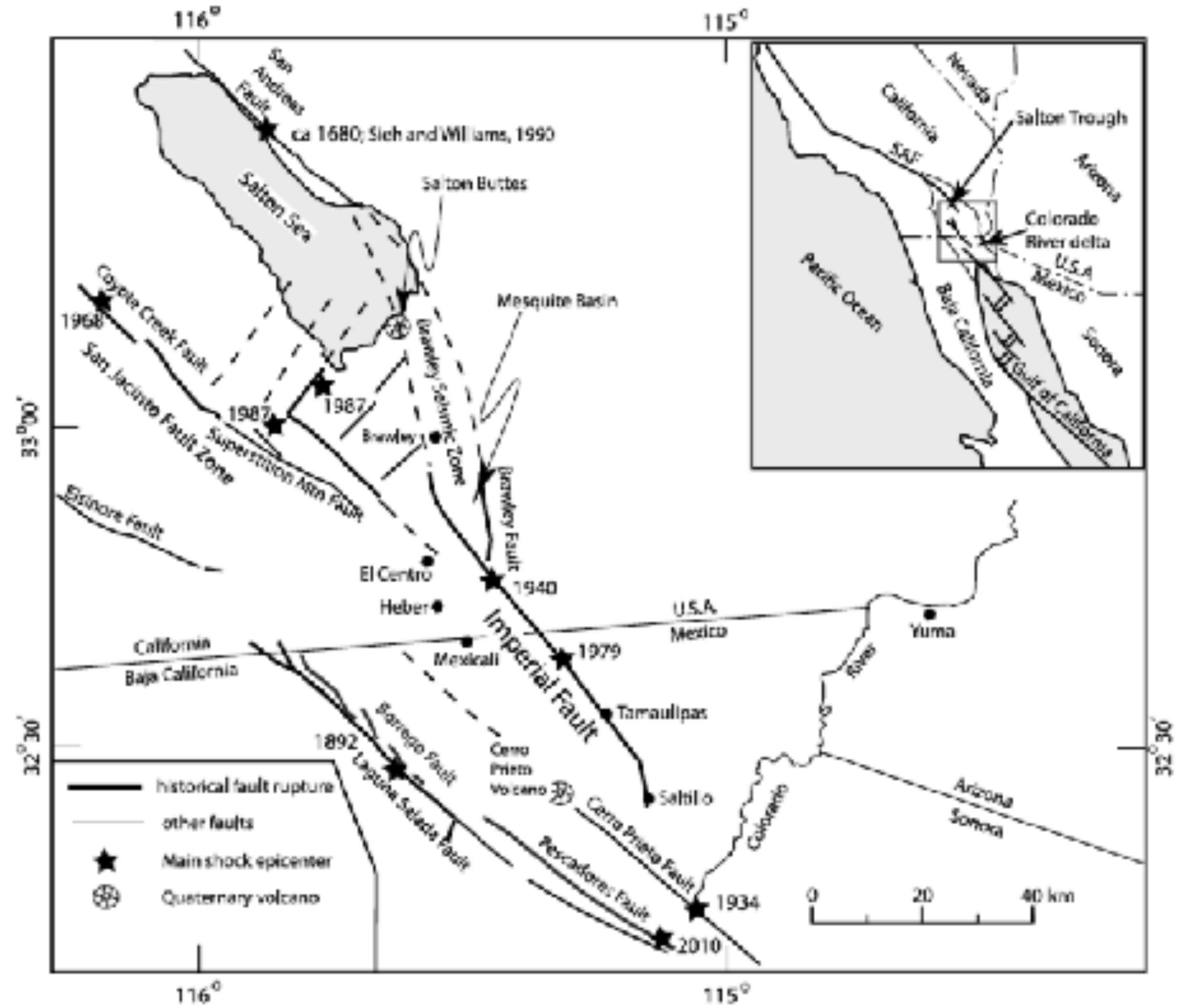
or (anisotropic)

$$0 = \frac{\partial}{\partial y}\left[\mu_1\frac{\partial u}{\partial y} + \mu_2\frac{\partial u}{\partial z}\right] + \frac{\partial}{\partial z}\left[\mu_3\frac{\partial u}{\partial y} + \mu_4\frac{\partial u}{\partial z}\right] \tag{2b}$$

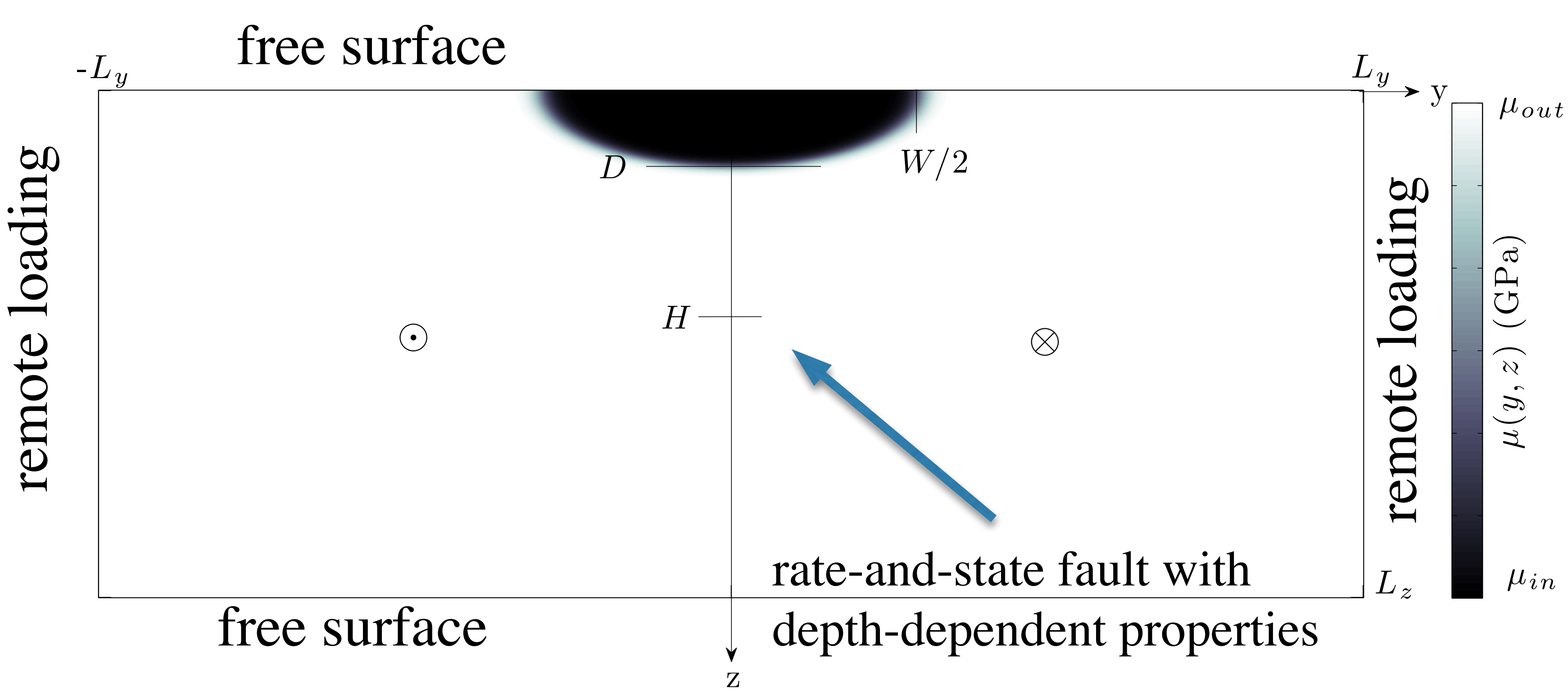
for out-of-plane displacement u and shear modulus μ .

Sedimentary Basins

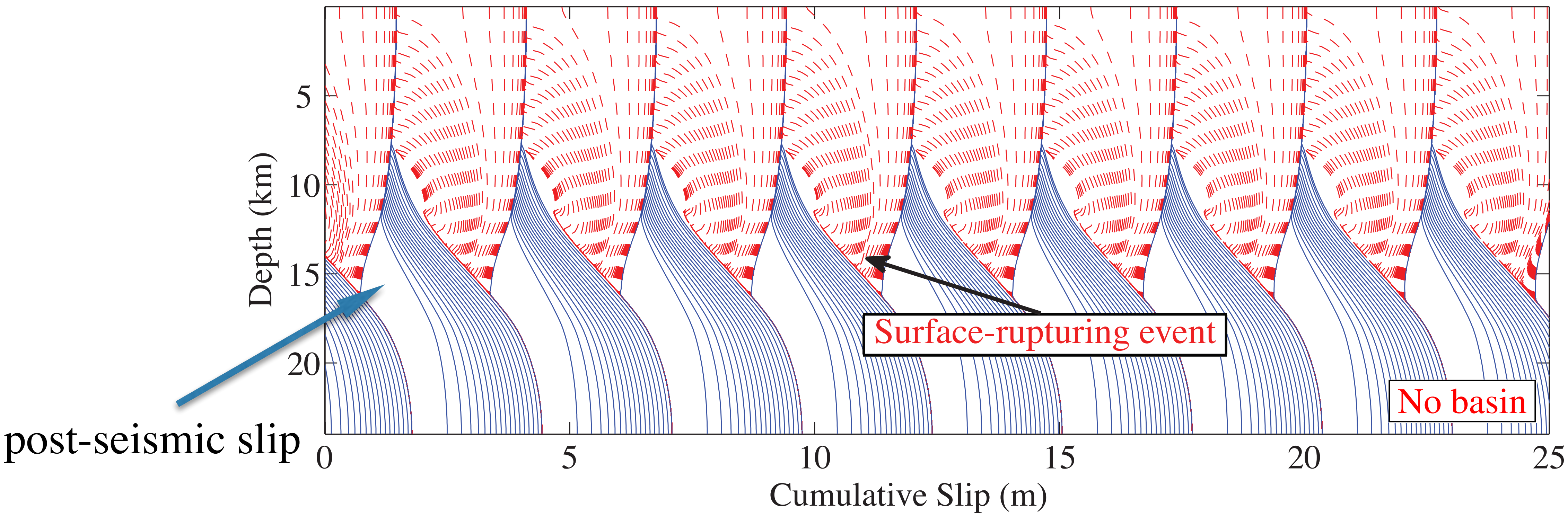
Map of the Imperial Valley Fault (Rockwell and Klinger, 2013). 1979 M6.5 event remained largely buried, vs 1940 M7 event featured extensive surface slip.



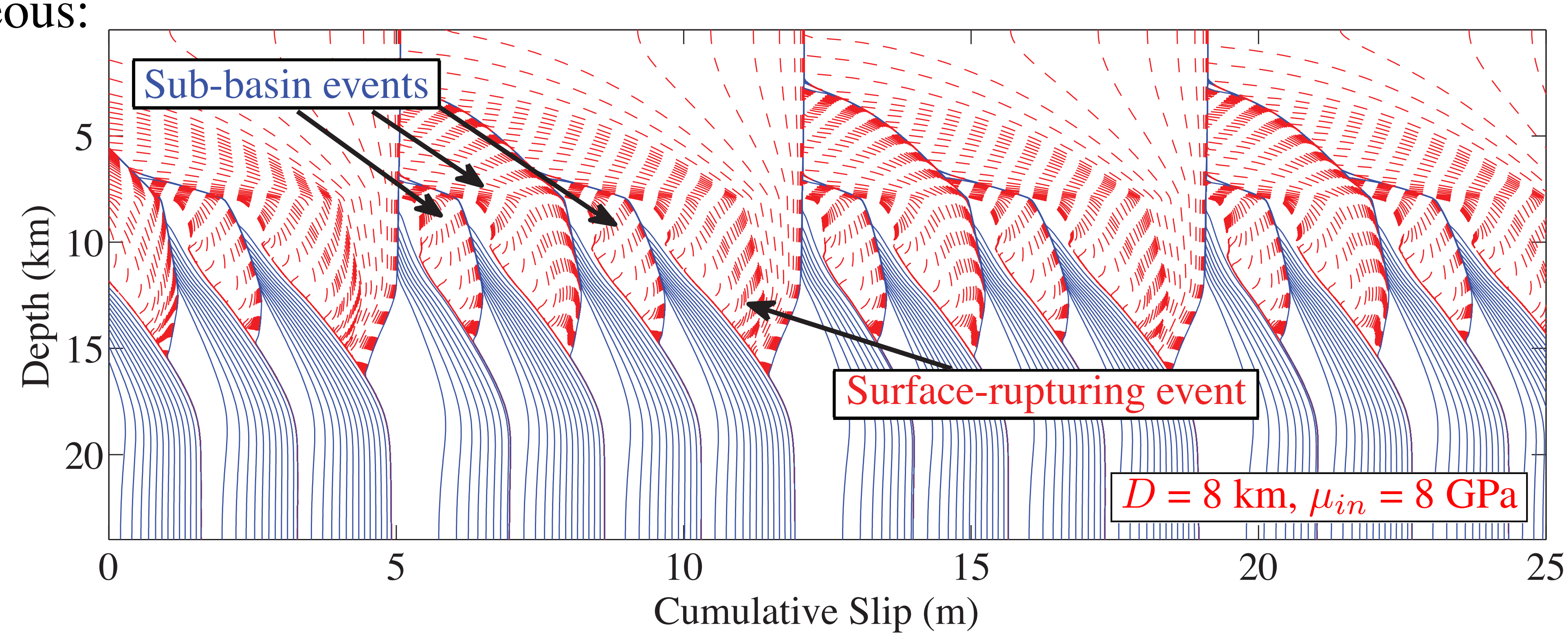
Schematic for model:



Homogeneous:

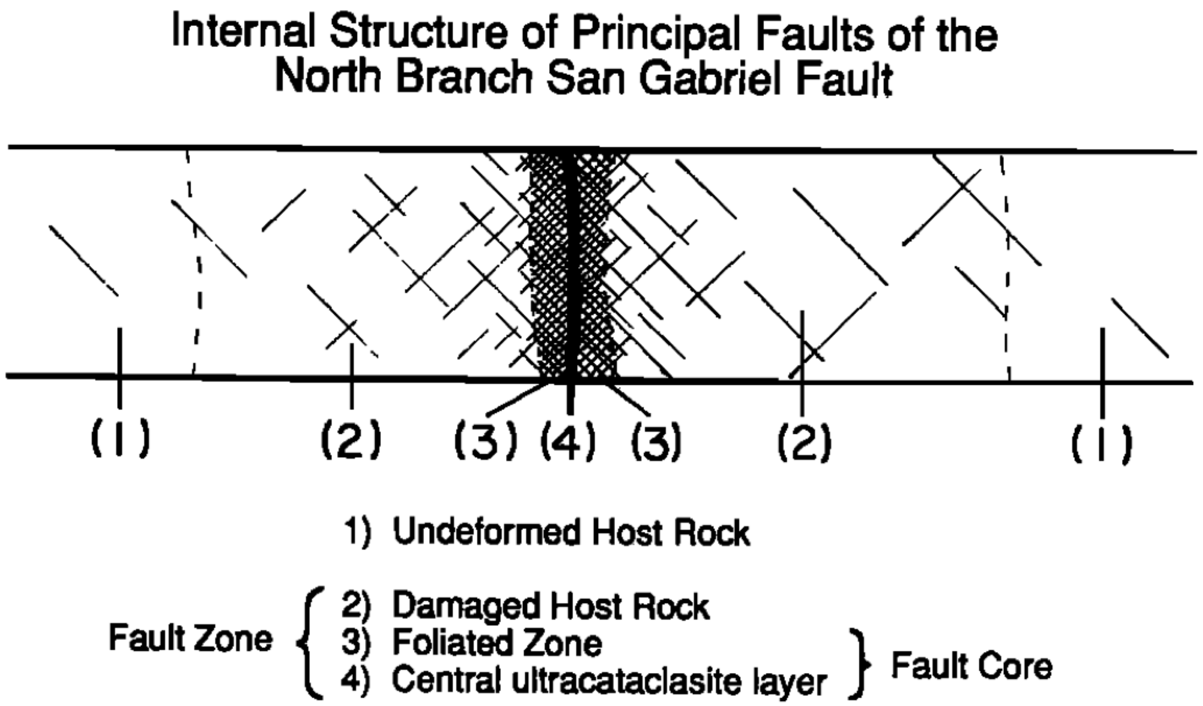


Heterogeneous:

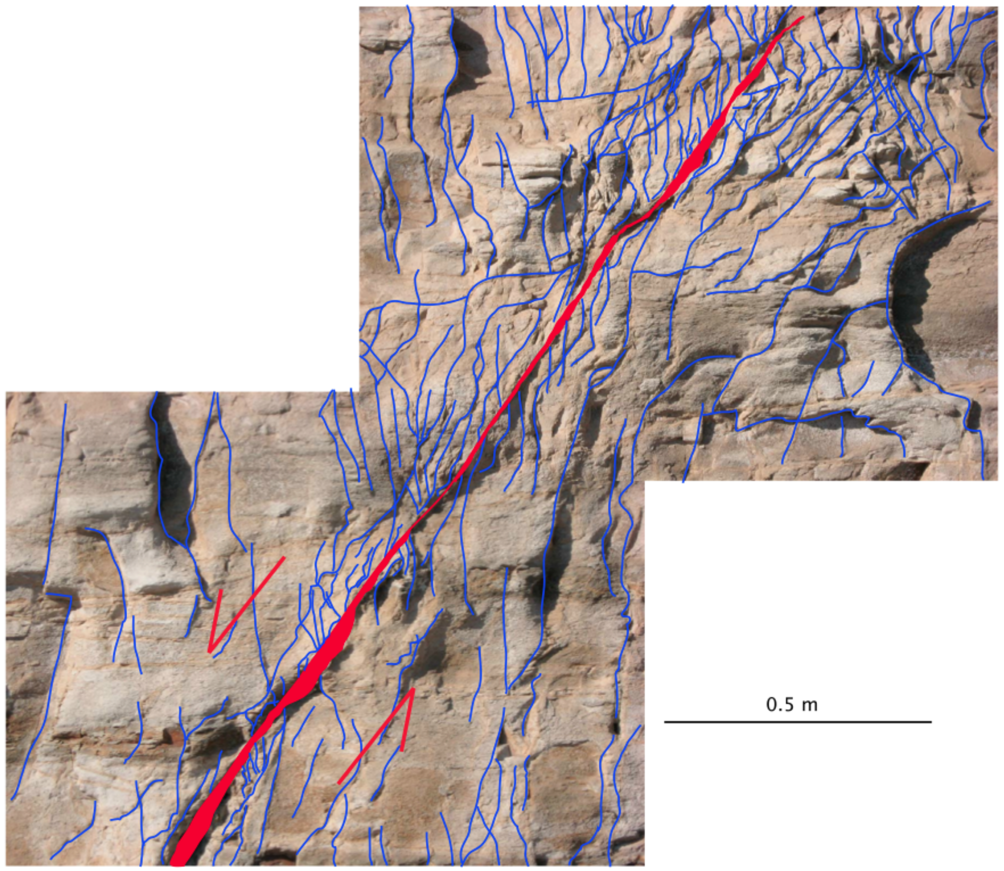


Slip profiles plotted in solid blue every 5 years during the interseismic period; in red every second during rupture. Sub-basin events leave a shallow slip deficit. Faults overlain with sediments can go unrecognized and can potentially host very large events.

Moving beyond the elastic assumption



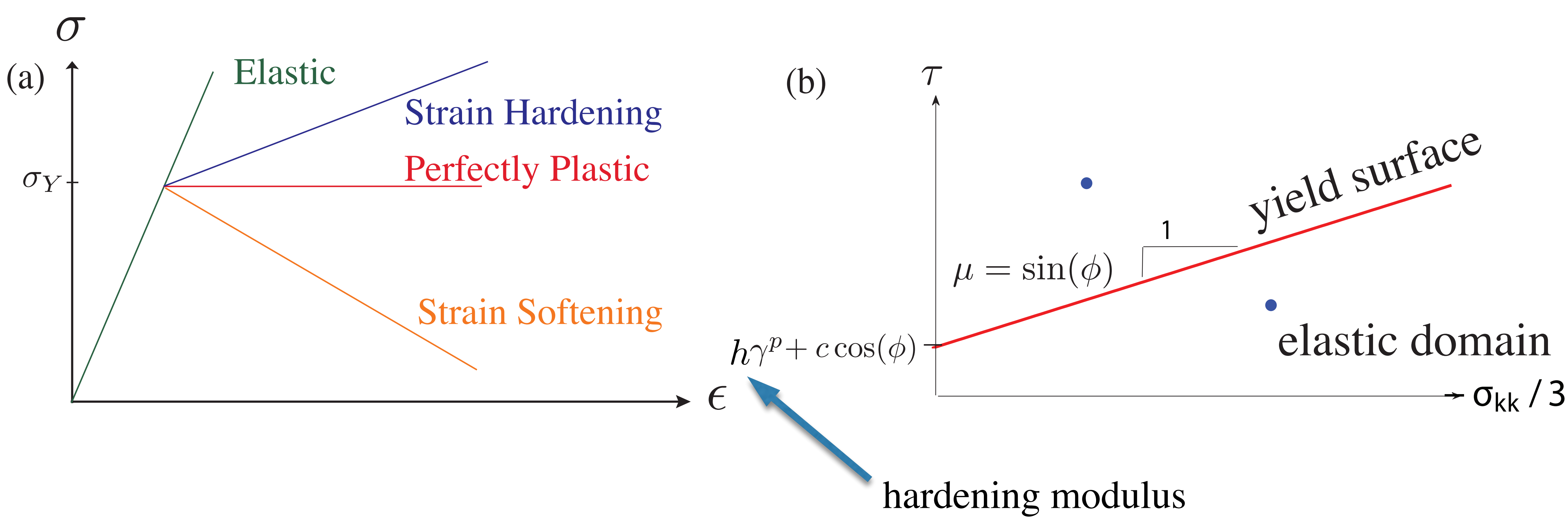
Chester et al. (1993)



Savage and Brodsky (2011)

Recent attention on the science of off-fault plasticity in order to understand the relationship between the degree of off-fault yielding and mechanical properties of fault zone material, how damage zones evolve with increasing cumulative slip and how these damage zones affect subsequent rupture and alter slip, recurrence intervals, surface deformation etc. How much tectonic off-set accommodated by plastic deformation? Can plastic behavior help explain discrepancies in geodetic data (e.g. Lindsey et al., 2014).

Constitutive Laws for Plasticity



Elastic domain

$$E_\sigma = \{\sigma : F(\sigma) \leq 0\}$$

for yield function F.

Constitutive Laws for Plasticity

Hooke's Law: $\sigma = C : (\epsilon - \epsilon^p)$ (3)

Flow rule: $\dot{\epsilon}^p_{ij} = \lambda P_{ij}(\sigma)$ (4)

for magnitude of plastic strain rate λ. P partitions plastic strain rate between components.

Kuhn-Tucker & persistency

$$\lambda \geq 0, \quad F \leq 0, \quad \lambda F = 0 \tag{5}$$

$$\lambda \dot{F} = 0 \tag{6}$$

Drucker-Prager Plasticity

Yield function: $F(\sigma, \gamma^p) = \bar{\tau} - (\sigma_Y + h\gamma^p)$

Flow rule: $\dot{\epsilon}^p_{ij} = \lambda \left[s_{ij}/2\bar{\tau} + (\beta/3)\delta_{ij} \right]$

- equivalent plastic strain γ^p
- deviatoric stress & second invariant $s, \quad \bar{\tau}$
- yield stress σ_Y
- hardening modulus h
- plastic dilatancy β

Equilibrium equation:

Hooke's law with elastoplastic tangent stiffness tensor

$$d\sigma = C^{ep}(\sigma) : d\epsilon \tag{7}$$

(Incremental) governing equations

$$0 = \nabla \cdot d\sigma \quad \rightarrow \quad 0 = \nabla \cdot (C^{ep}(\sigma) : d\epsilon) \tag{8}$$

For antiplane motion: a second order, nonlinear elliptic PDE for the displacement increment du.

For D-P rate-independent plasticity, this reduces to:

$$0 = \frac{\partial}{\partial y}\left[C^{ep}_{yy}\frac{\partial du}{\partial y} + C^{ep}_{yz}\frac{\partial du}{\partial z}\right] + \frac{\partial}{\partial z}\left[C^{ep}_{zy}\frac{\partial du}{\partial y} + C^{ep}_{zz}\frac{\partial du}{\partial z}\right] \tag{9}$$

Where the specifics components are, for example,

$$C^{ep}_{yy}(\sigma) = \begin{cases} \mu & \text{if } \lambda = 0, \\ \mu - \frac{\mu\sigma^2_{xy}/\bar{\tau}^2}{1+h/\mu} & \text{if } \lambda > 0 \end{cases}$$

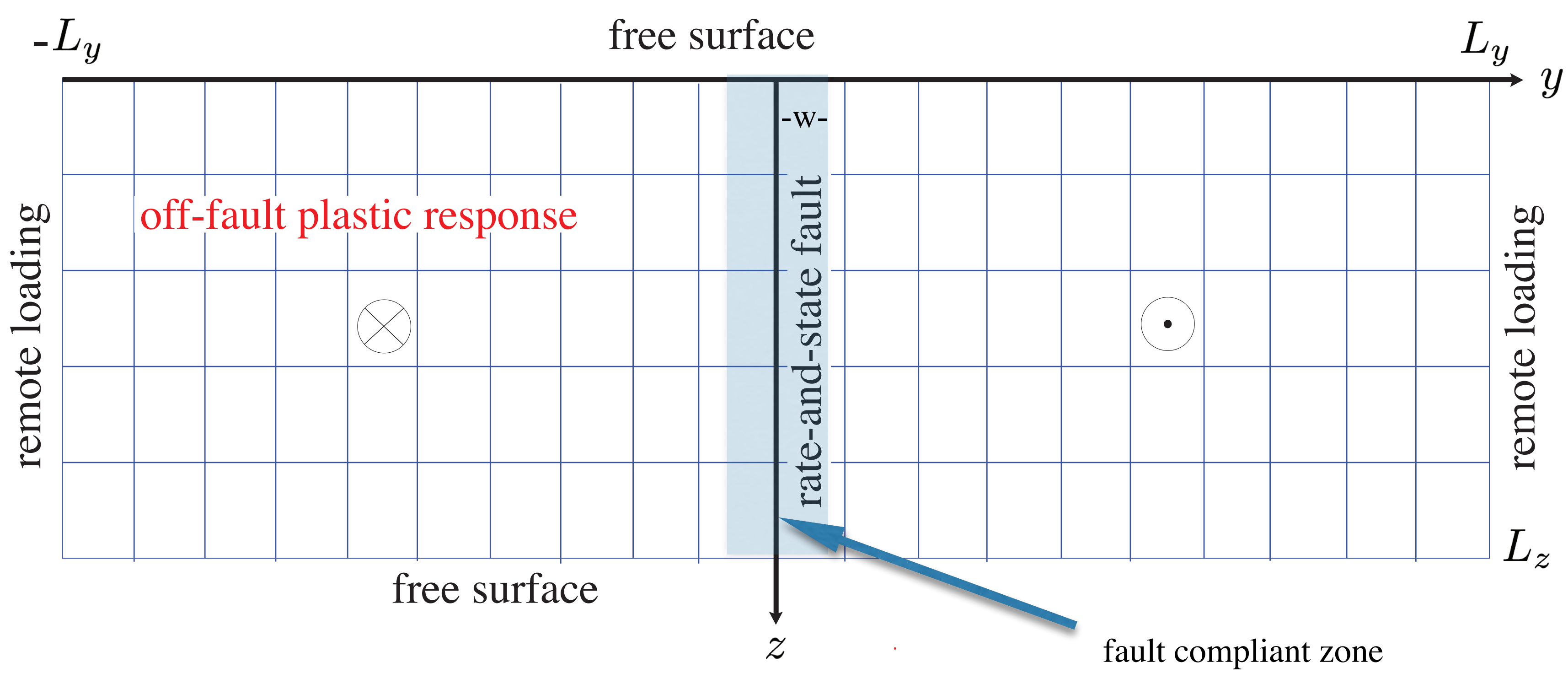
(9) maintains ellipticity during plastic flow iff

$$C^{ep}_{yy}C^{ep}_{zz} - \left[C^{ep}_{yz}\right]^2 = \frac{\mu h}{1+h/\mu} > 0$$

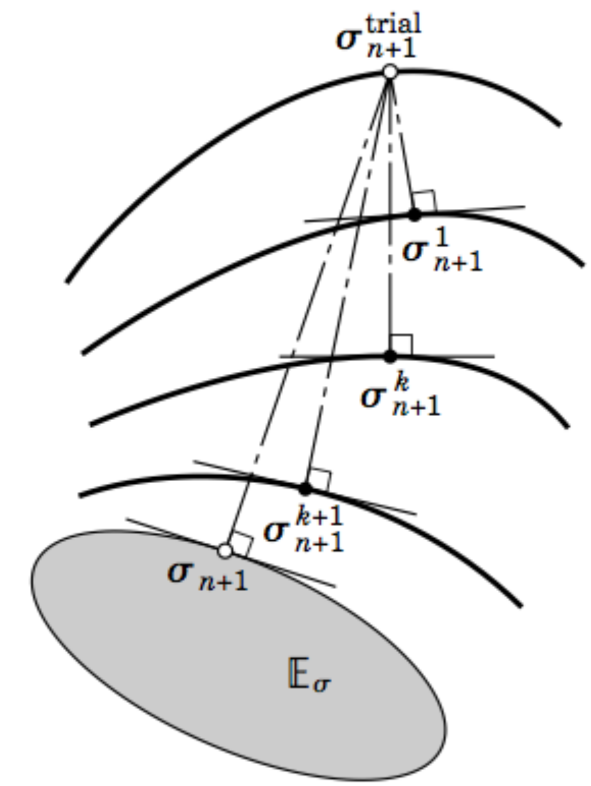
thus, hardening must be present.

Methodology

Schematic:



Return-mapping algorithm:

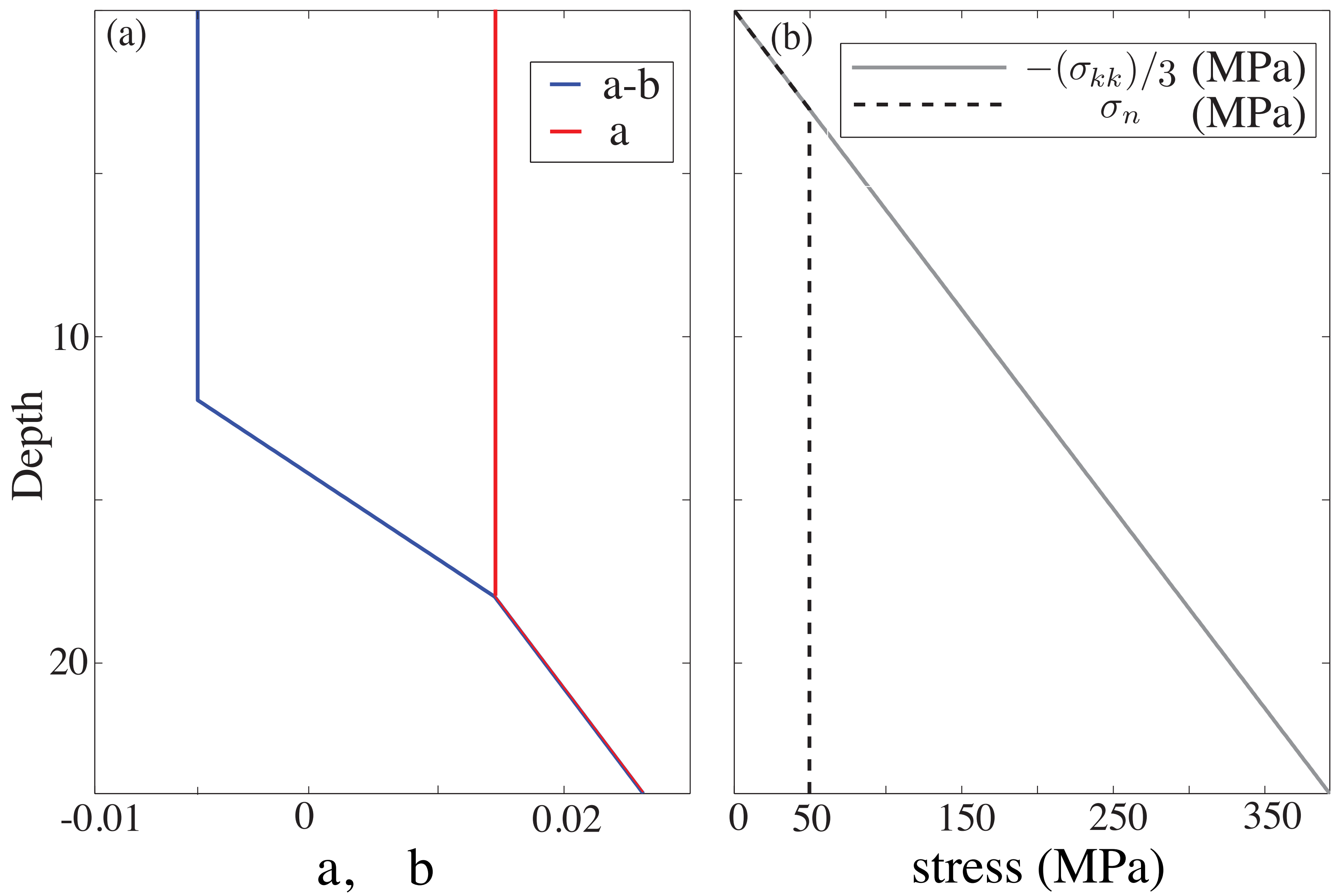


- Spatial discretization with a finite difference method.

- Time stepping: stresses, strains, and displacements are updated by solving equilibrium with an iterative Newton method together with boundary conditions that impose slow, tectonic loading and slip (in a manner consistent with rate-and-state friction), and the return-mapping algorithm for consistent stresses.

Rate-and-state friction:

$$\tau = \sigma_n f(V,\psi), \quad f(V,\psi) = a \sinh^{-1}\left(\frac{V}{2V_0} e^{\psi/a}\right)$$

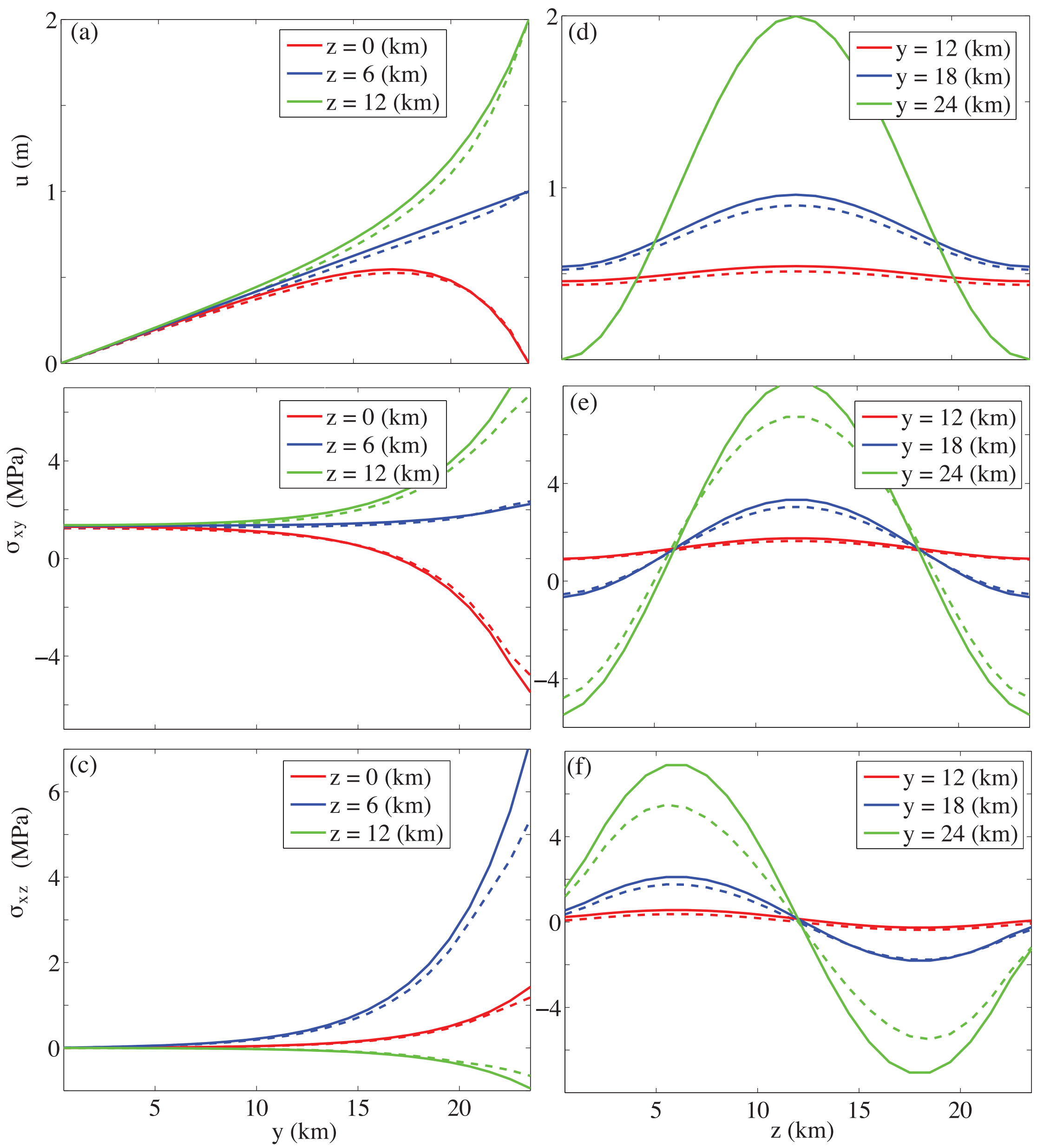


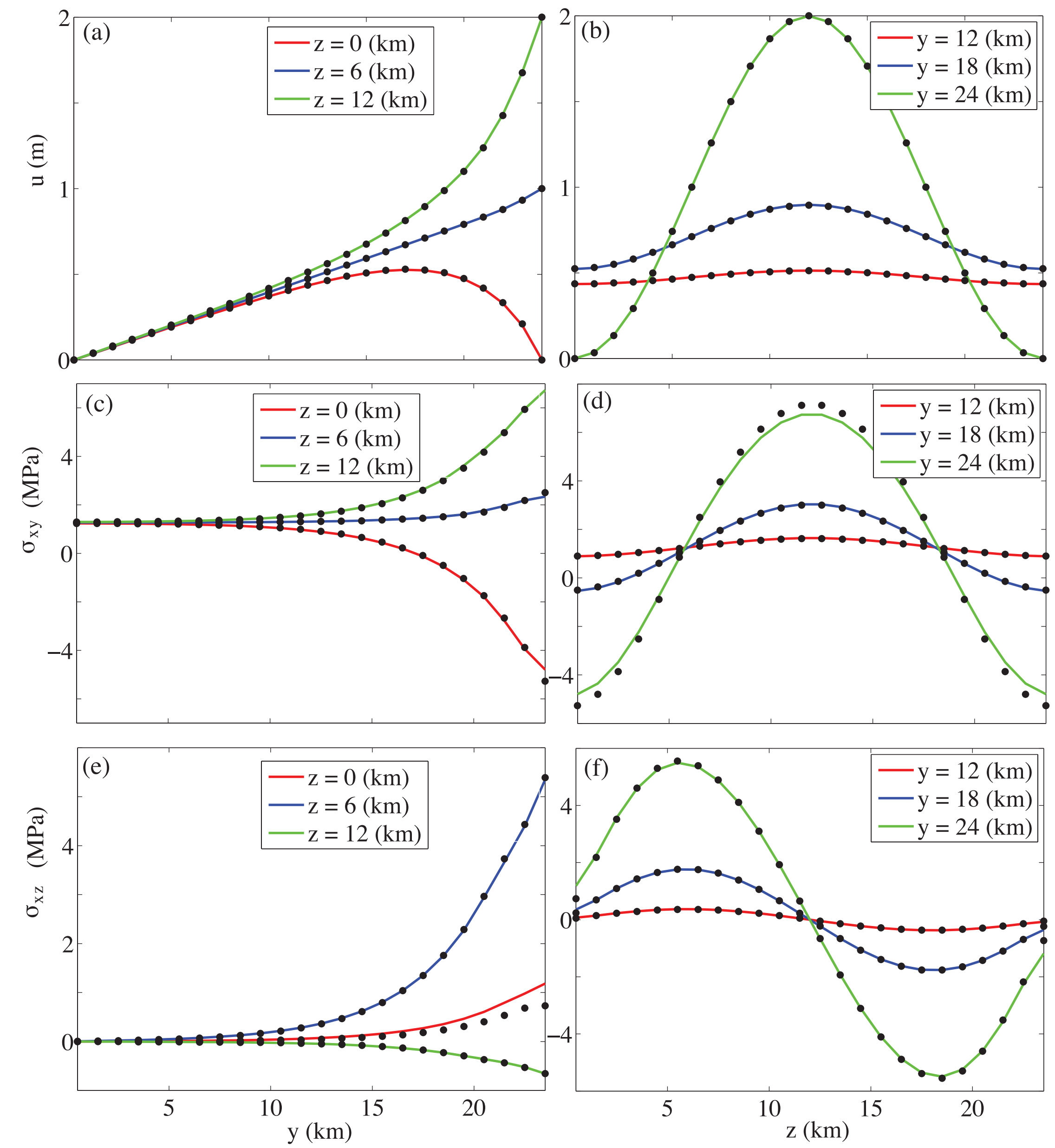
Rate-and-state parameters and depth-dependent yield stress $\sigma_Y = -(\sigma_{kk}/3)\sin(\phi) + c\cos(\phi)$ for cohesion c and internal friction angle ϕ.

Verification Studies

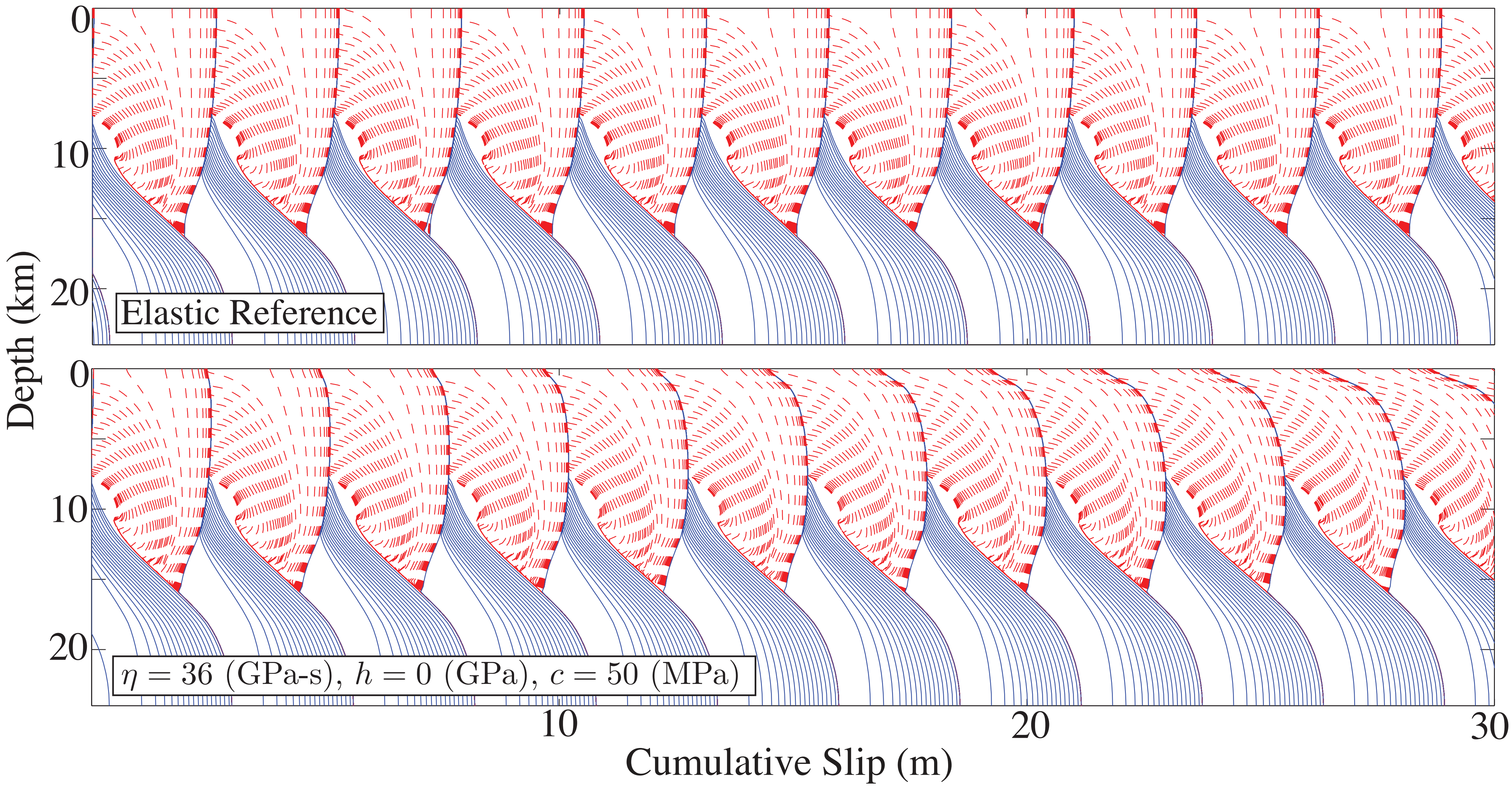
Study 1: manufactured solution to elastic, anisotropic problem, with time stepping and r-s friction.

Study 2: boundary value plastic-problem, comparison with FEM solution from OpenSees (<u>http://opensees.berkeley.edu</u>), right Figure below (FEM solution in black dots).



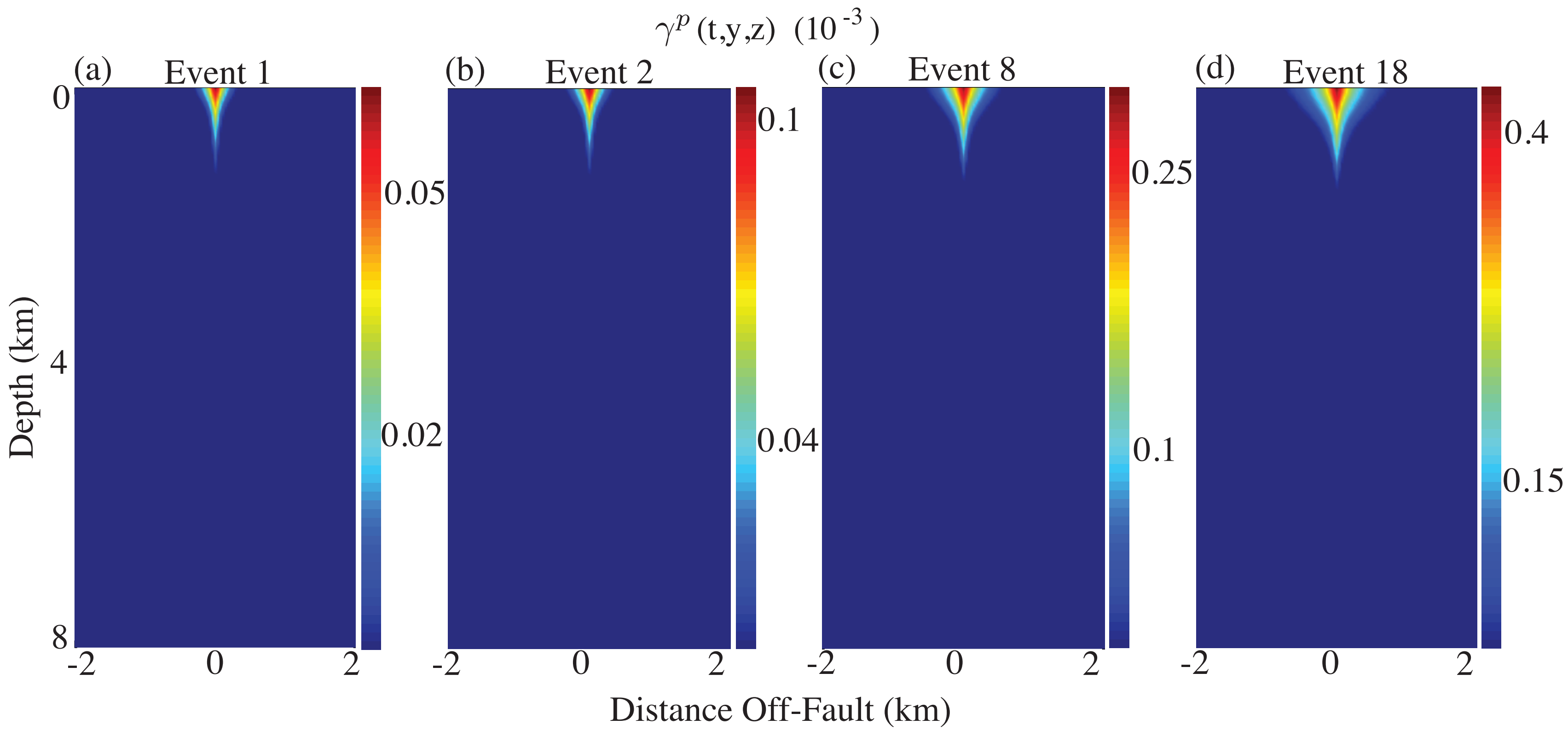


Earthquake cycles with plastic response

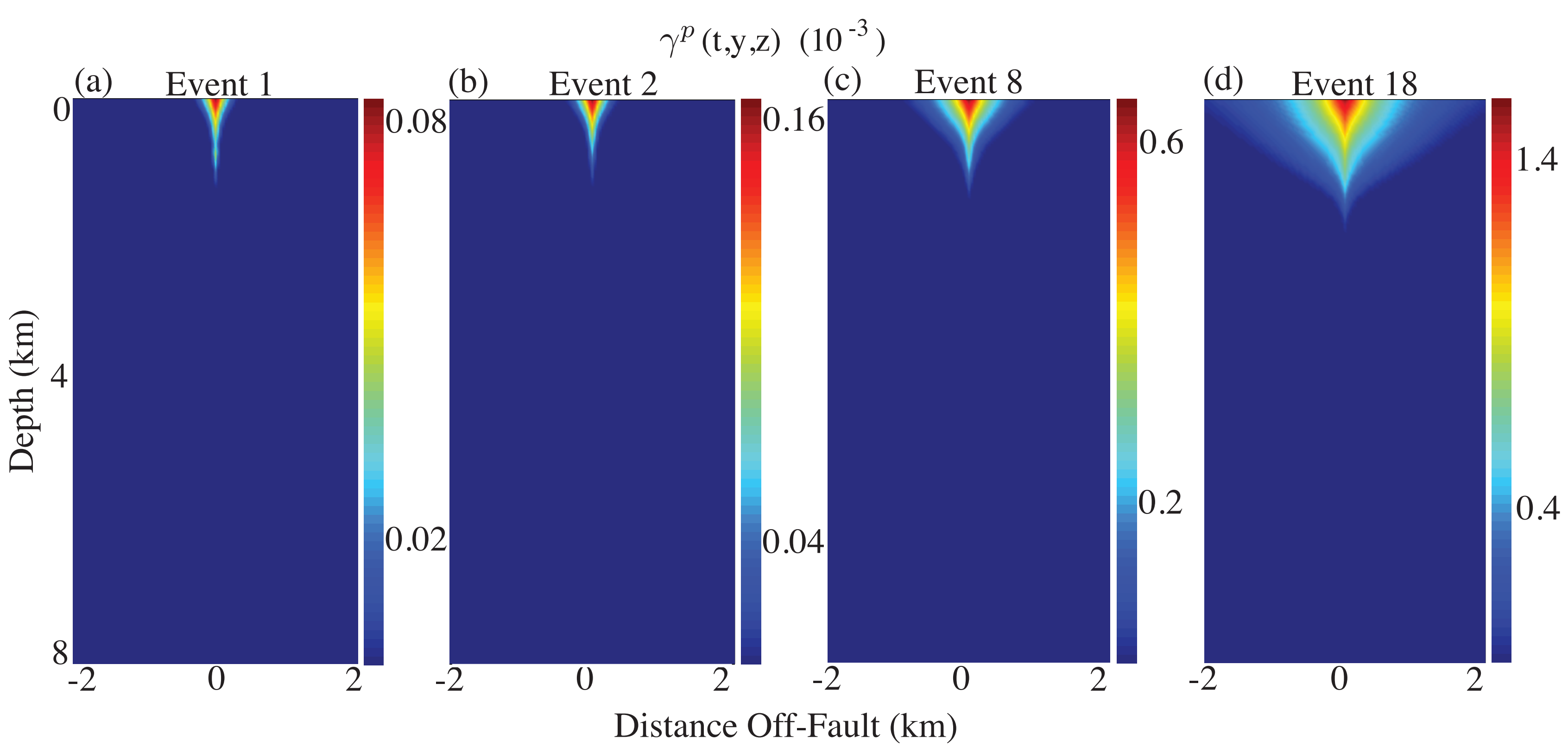


Slip profiles plotted in solid blue every 5 years during the interseismic period; in red every second during rupture.

Magnitude and extent of off-fault plastic strain



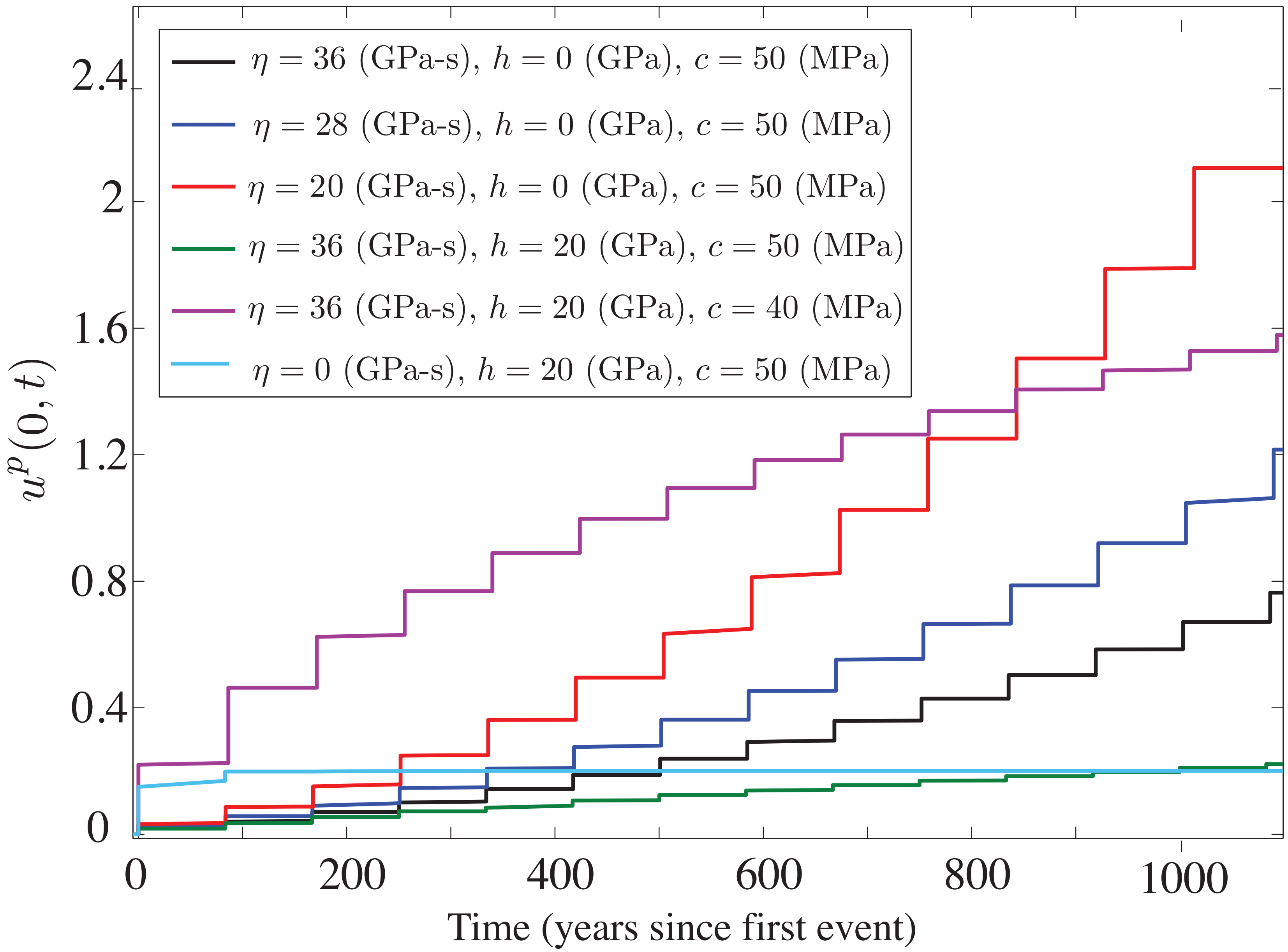
Viscoplastic with hardening: extent and magnitude saturate.



Viscoplastic without hardening: no saturation (extent increases ~100 m per rupture).

How much off-set is accommodated by plastic strain?

$$u^p(t, z) = \int_{-L_y}^{L_y} \gamma^p_{xy}(t, y, z)\, dy$$

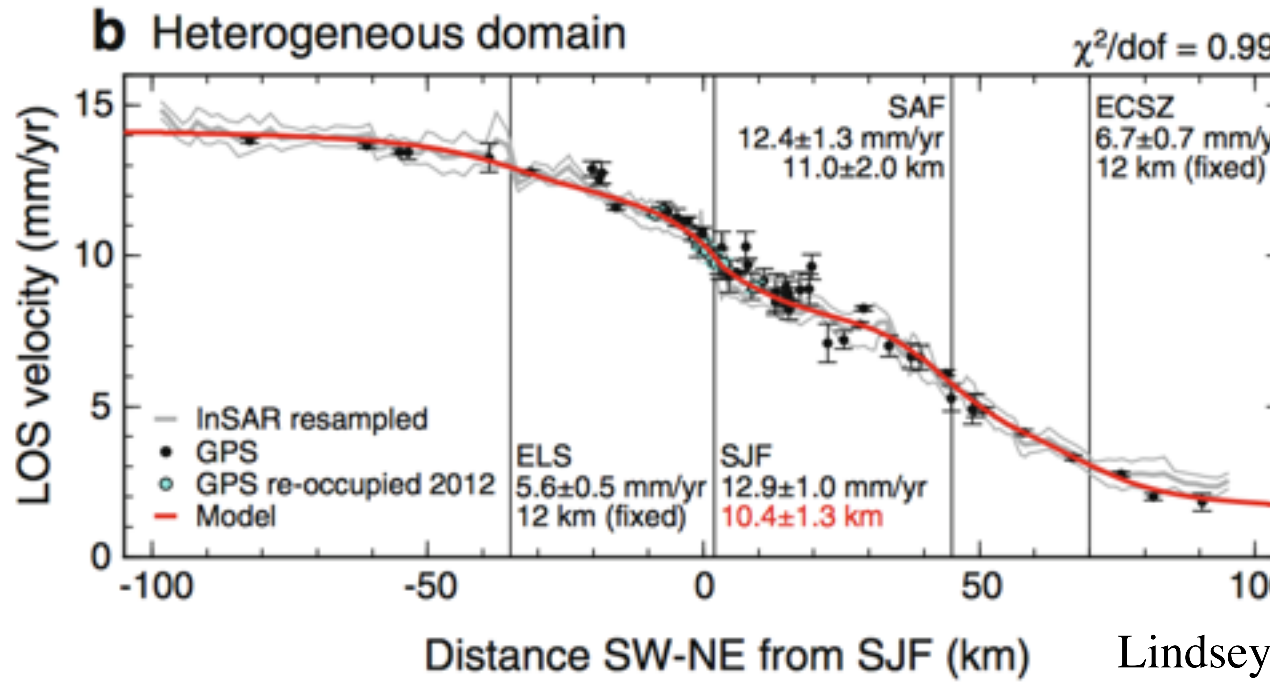


Most significant amount for viscoplastic models without hardening (~2 m per 10 ruptures, 10% of tectonic deformation budget). If SSD deficit of 3-19% exists (Xu et al., 2016), then some of this can be attributed to plastic deformation.

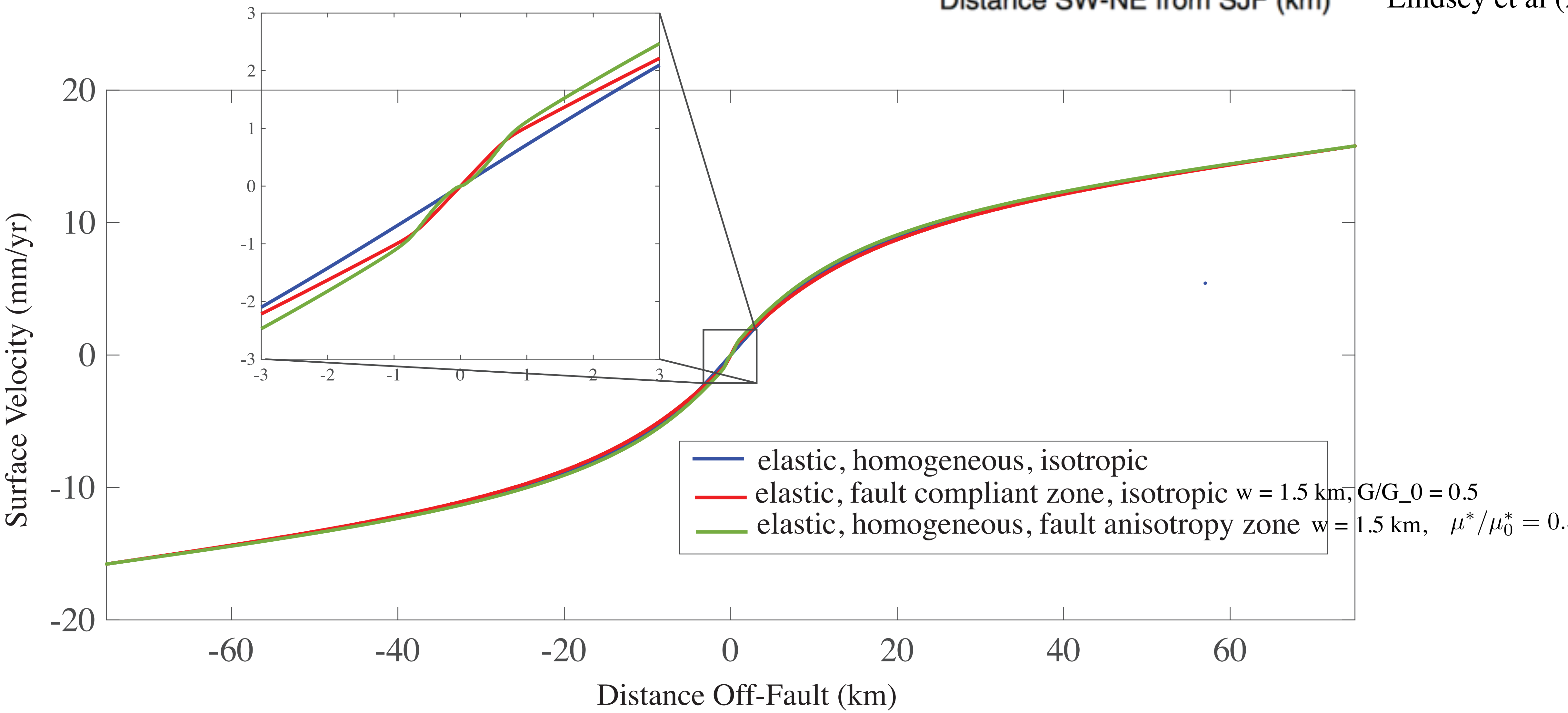
Interseismic surface deformation

*Plasticity problematic (at this point)

viscoplastic time-scale: η/μ



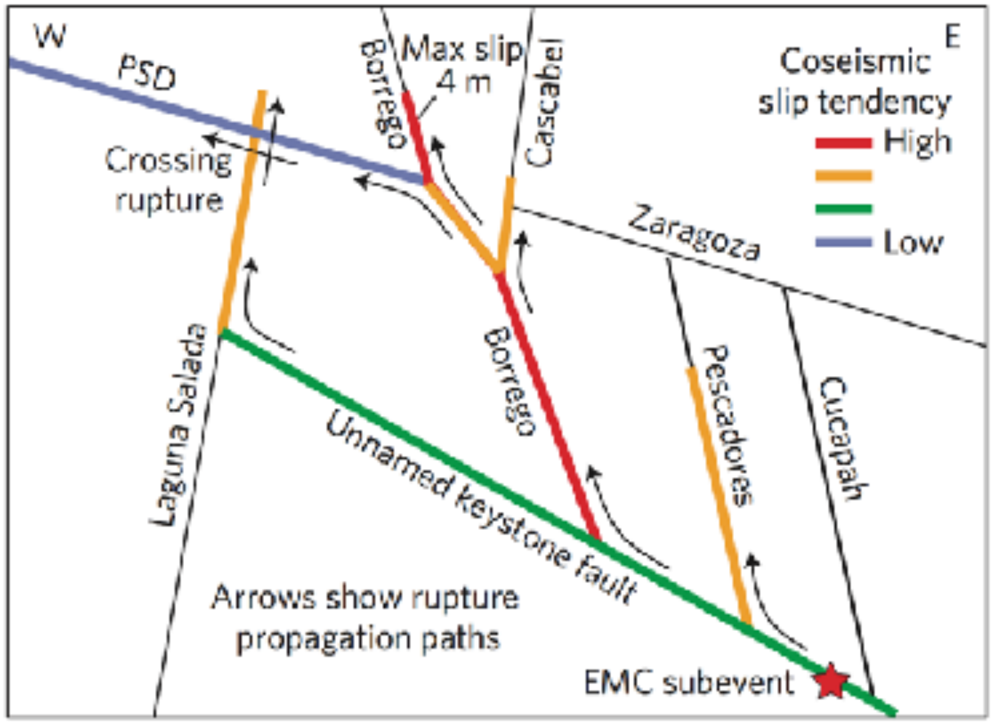
Lindsey et al (2014)



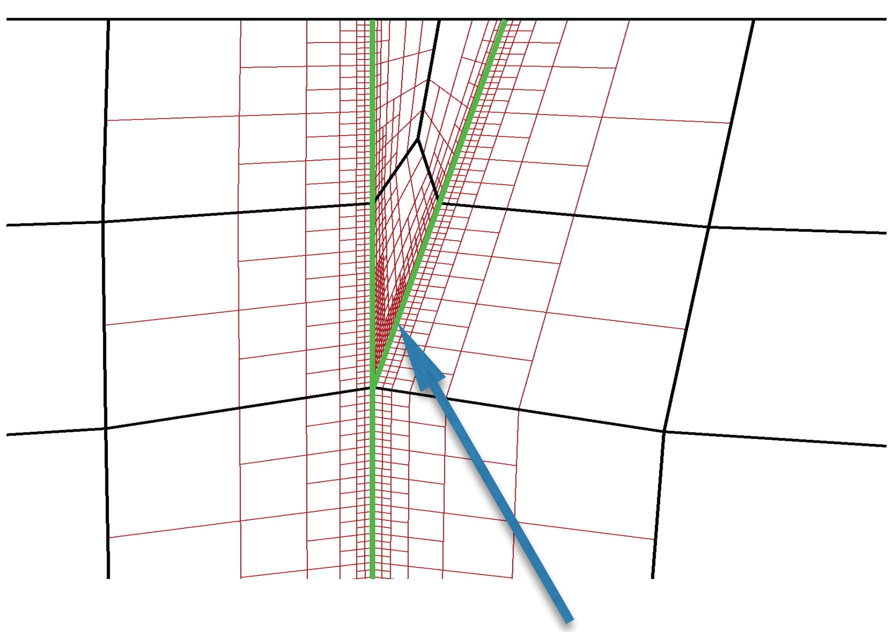
Discussion and current work

- Detailed parameter study of earthquake cycles with rupture into shallow sediments with off-fault plasticity. Looking towards comparison of model results with observables.

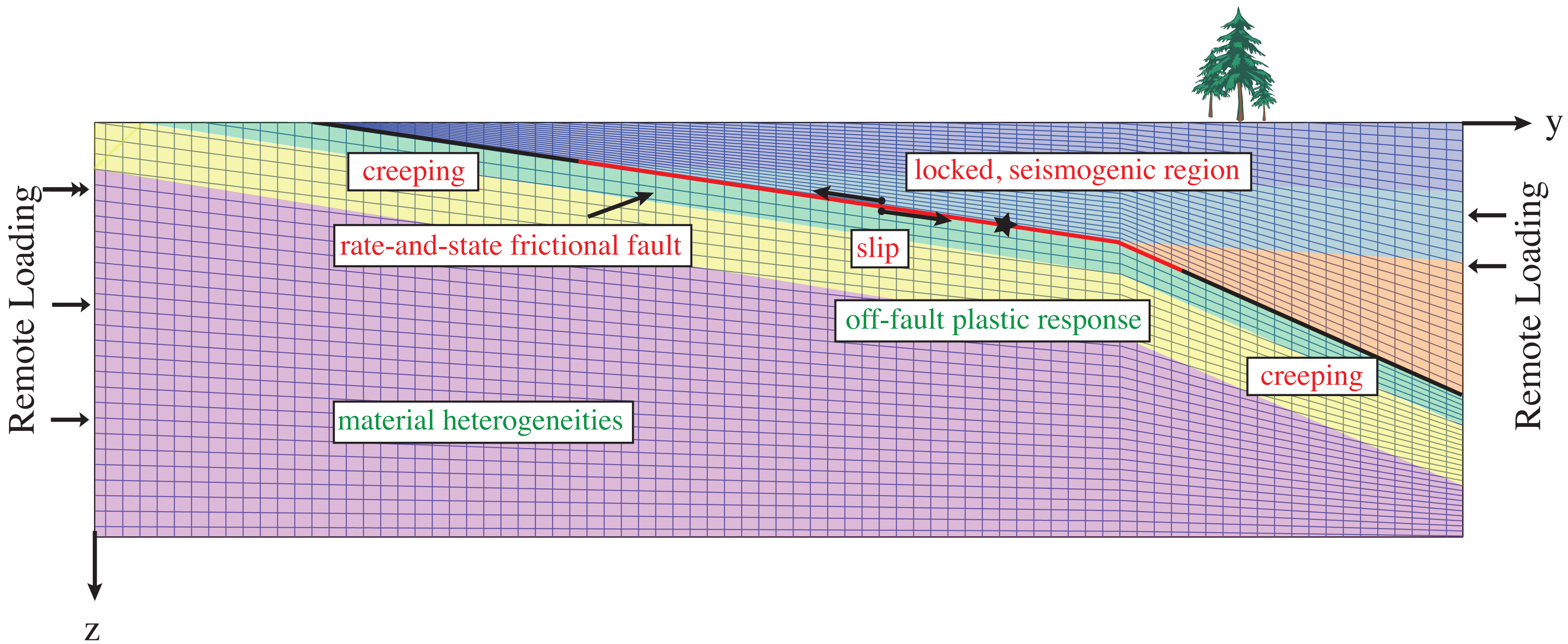
- With Jeremy Kozdon (Naval Postgraduate School). We are developing a new modeling framework to simulate cycles (with full dynamics) in complex geometries, based on DG with hp-adaptivity.



2010 El Mayor-Cucapah (EMC), Fletcher et al. (2016)



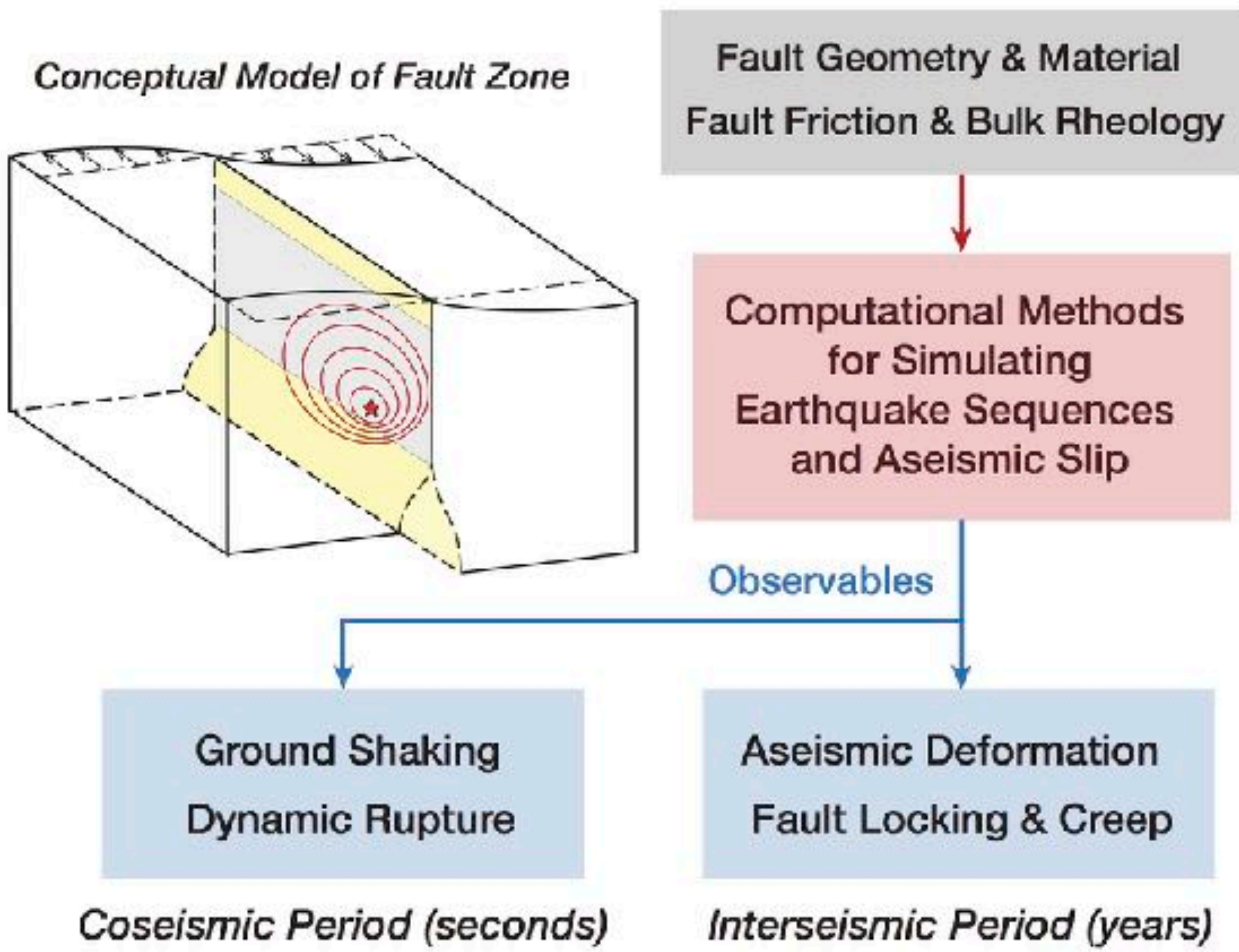
fault branch



Cycles in subduction zones

Collaborative Effort

TAG proposal in review with SCEC (16 co-PIs) in order to:



- understand how earthquakes nucleate, propagate and terminate by developing models that can simulate the slow, interseismic period between earthquakes, dynamic rupture, and afterslip, all of which are characterized by widely varying scales.
- discuss and further advance our computational capabilities, to promote robust results and reproducible science, and to lend experience and verifiable tools for newcomers to the field.

Please contact me if you are interested in being a part of this group: berickson@pdx.edu

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Coseismic surface deformation