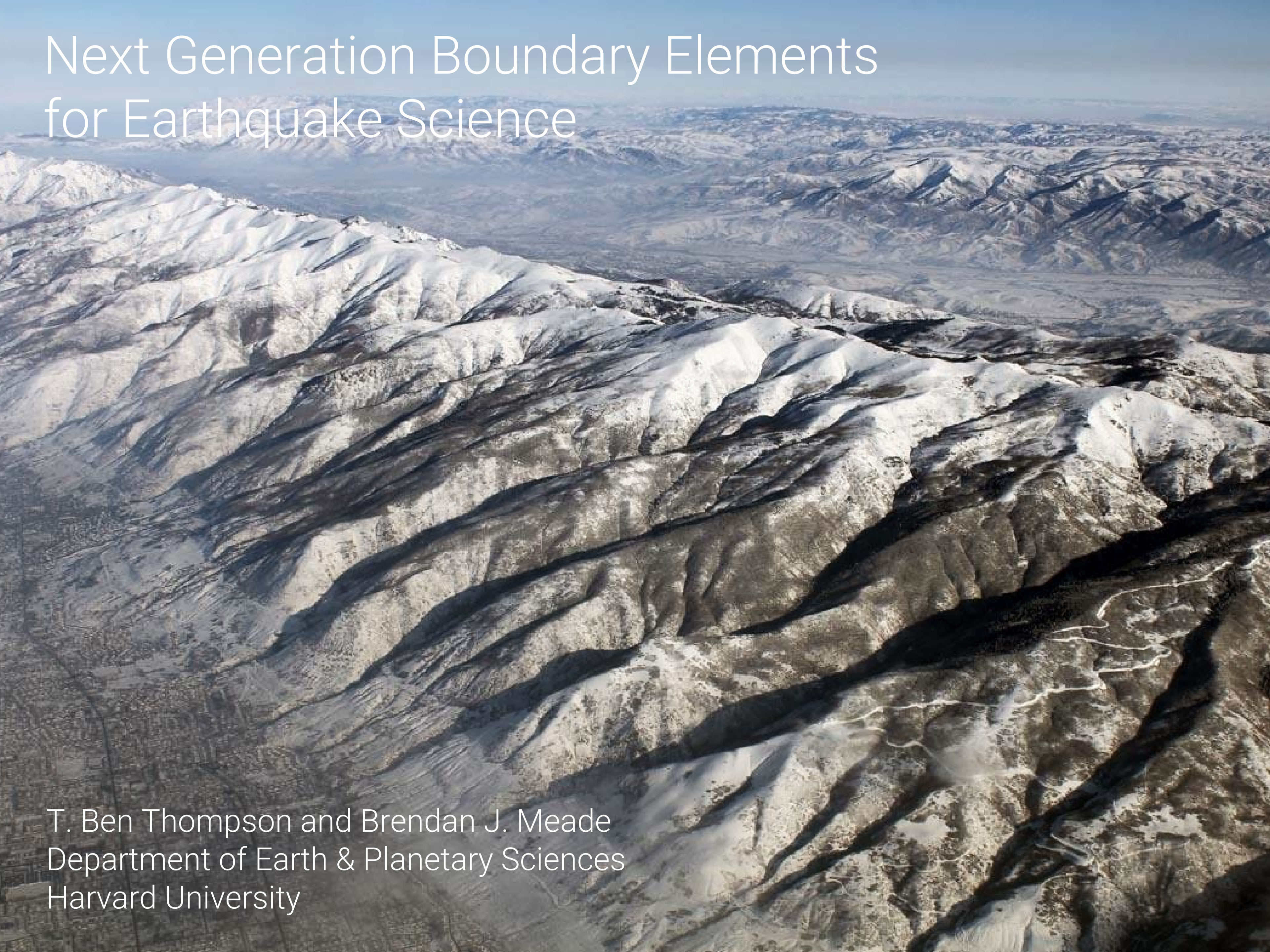
Next Generation Boundary Elements for Earthquake Science



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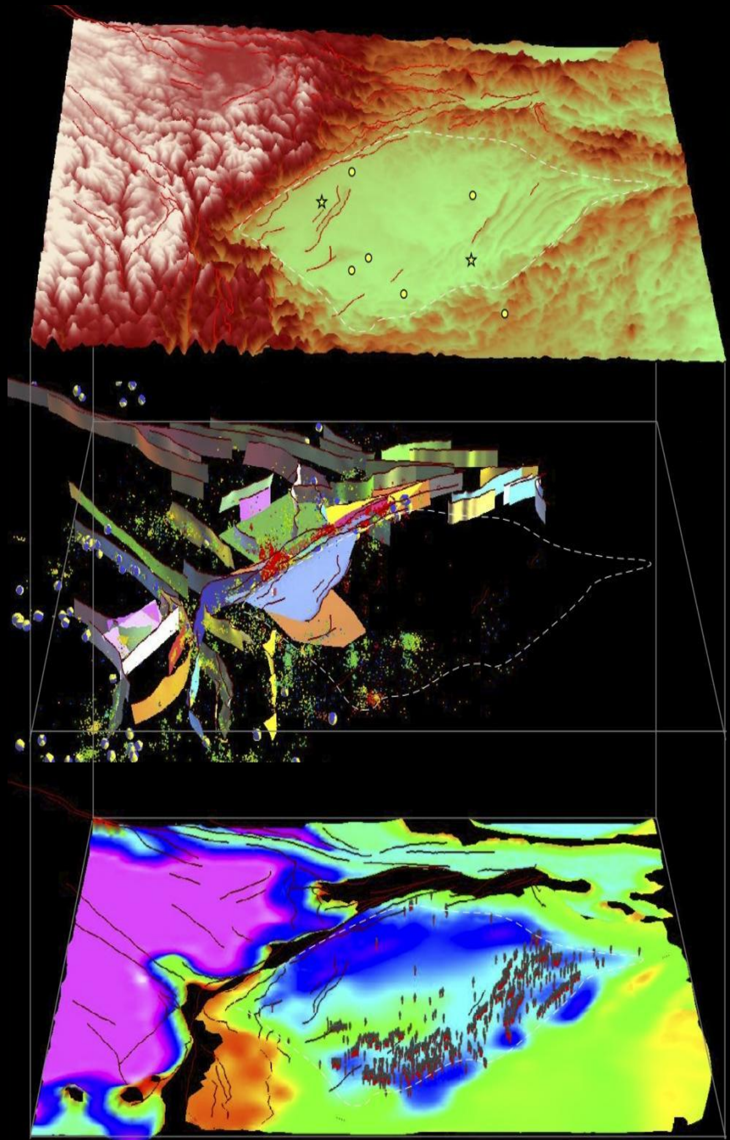
Harvard University

A fault/earthquake cycle model wish list:



Hubbard et al.

topography

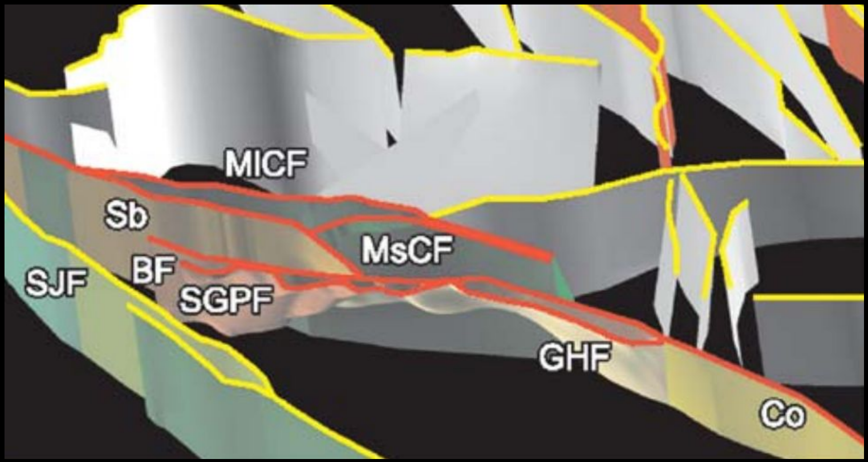
complex fault systems

material property variations

Better tools can speed up science

FEM requires a volumetric mesh

Incredibly difficult for realistic fault system geometries!

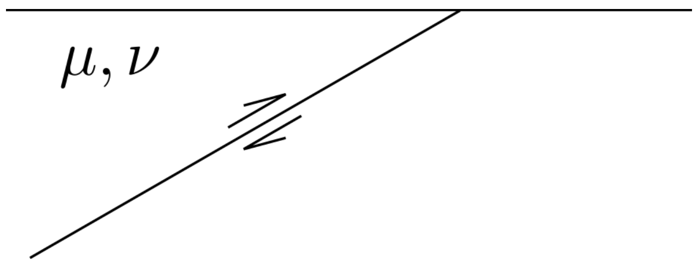


Plesch et al. 2007

"**80%** of overall analysis time is devoted to mesh generation"
-- Hughes et al. 2005

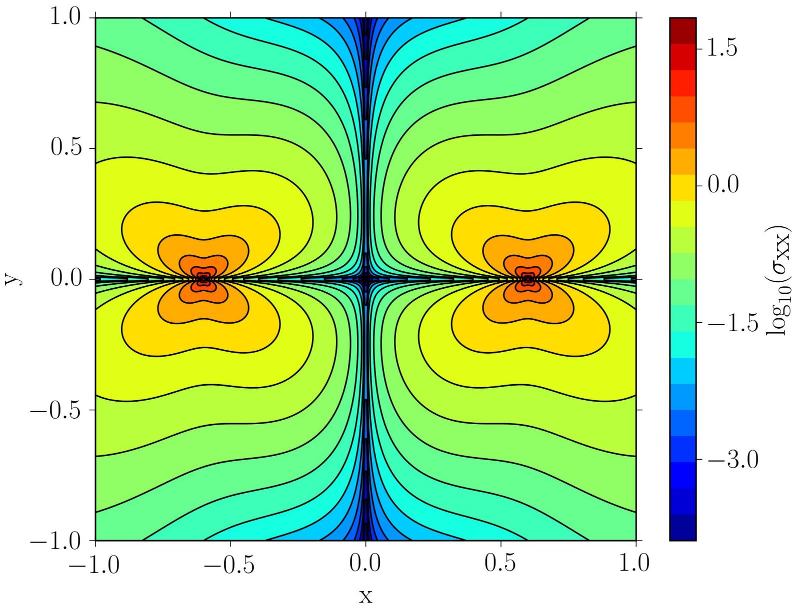
Current boundary element methods are also limited

Simple models



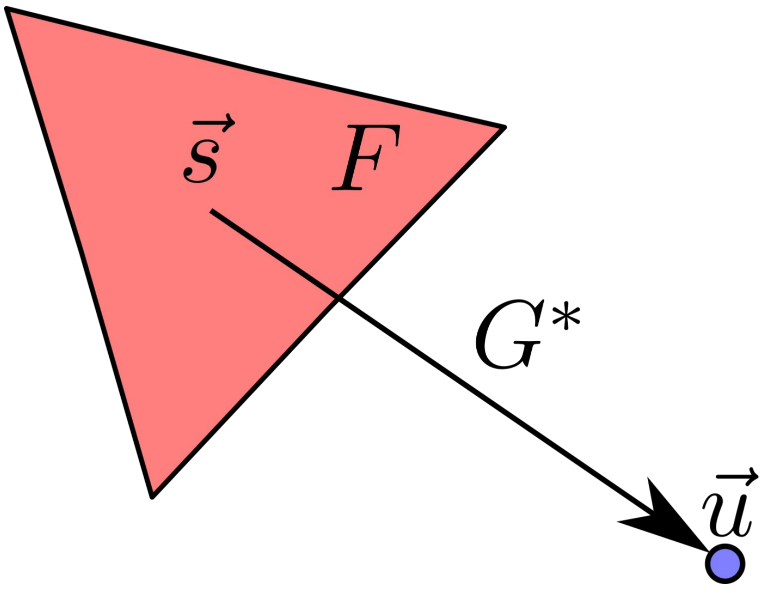
Small models (**<50k** elements)

Constant slip Okada dislocations produce unphysical singular stresses



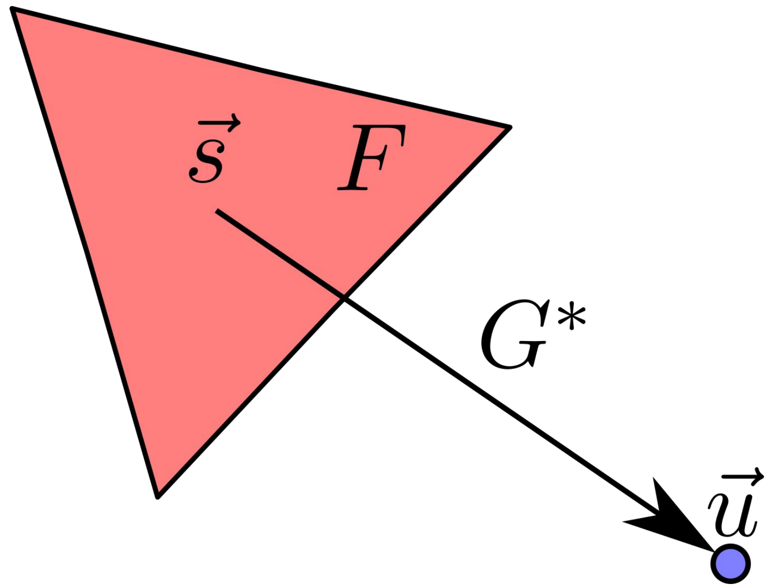
How are dislocation solutions derived?

$$\vec{u} = \int_F G^* \vec{s}\, dF$$



Integrate by hand:

$$\vec{u} = \int_F G^* \vec{s}\, dF$$



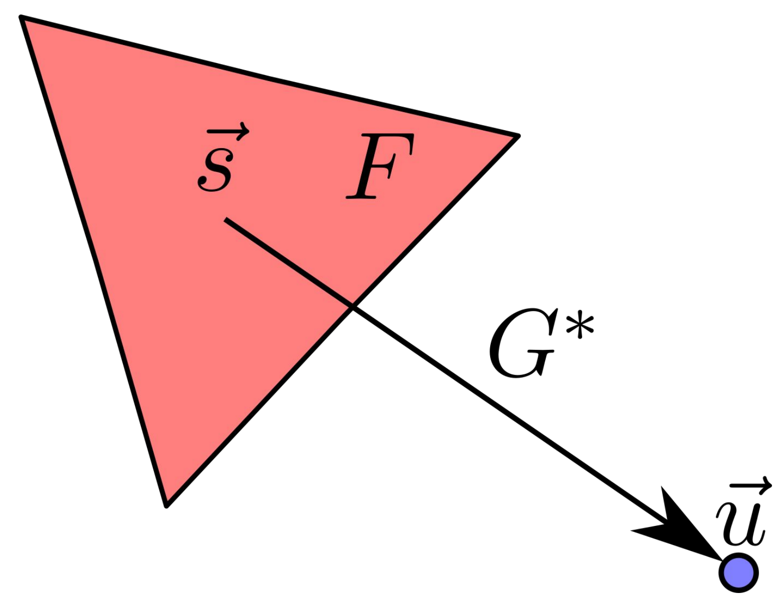
Assume:

point source slip Green's function
constant slip
a half-space

BEM limitations are due to analytical integration…
and are avoidable!

Numerically integrate!

$$\vec{u} = \int_F G^* \vec{s}\, dF$$



Any Green's function

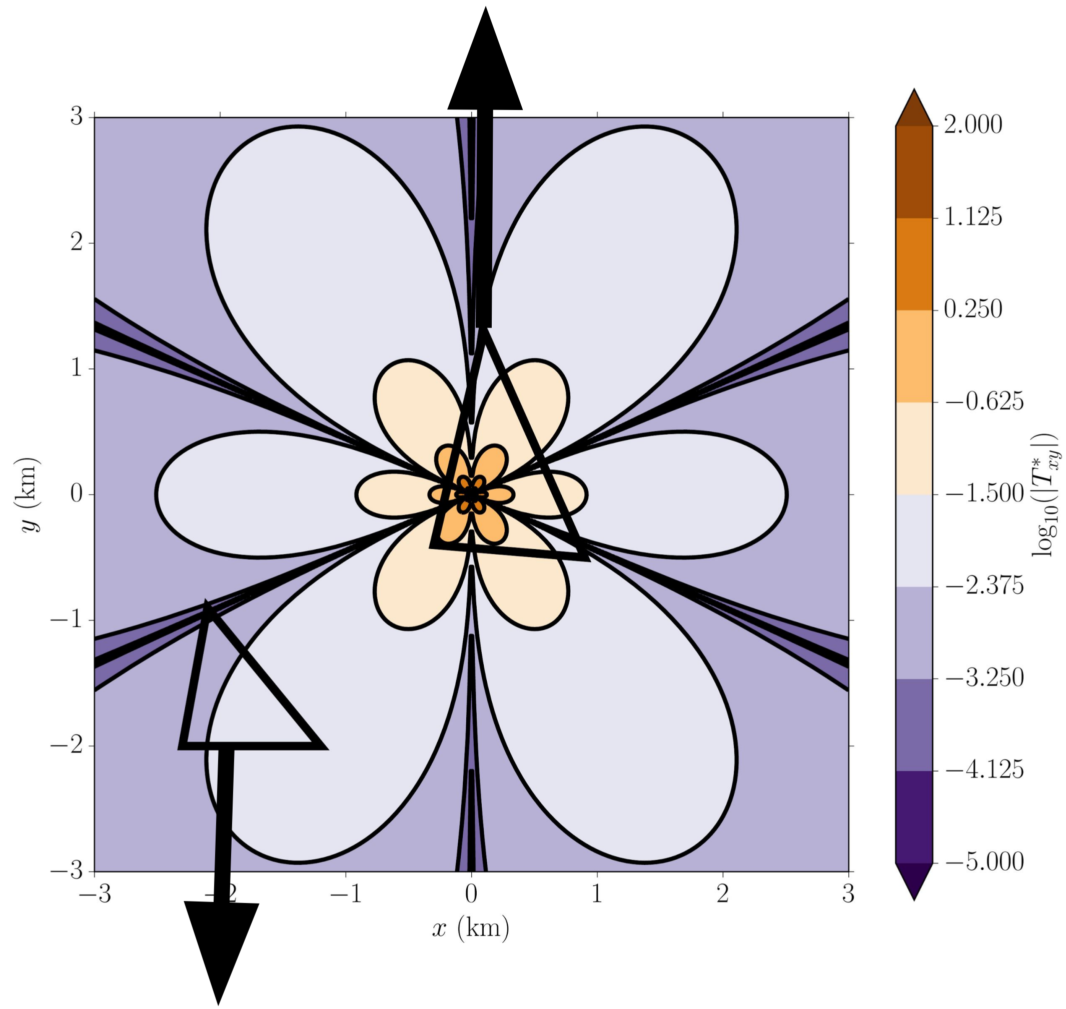
$\longrightarrow$ topography, material contrasts, gravity
(even viscoelasticity, Stokes flow)

Any basis for the slip

$\longrightarrow$ linear to avoid stress singularities

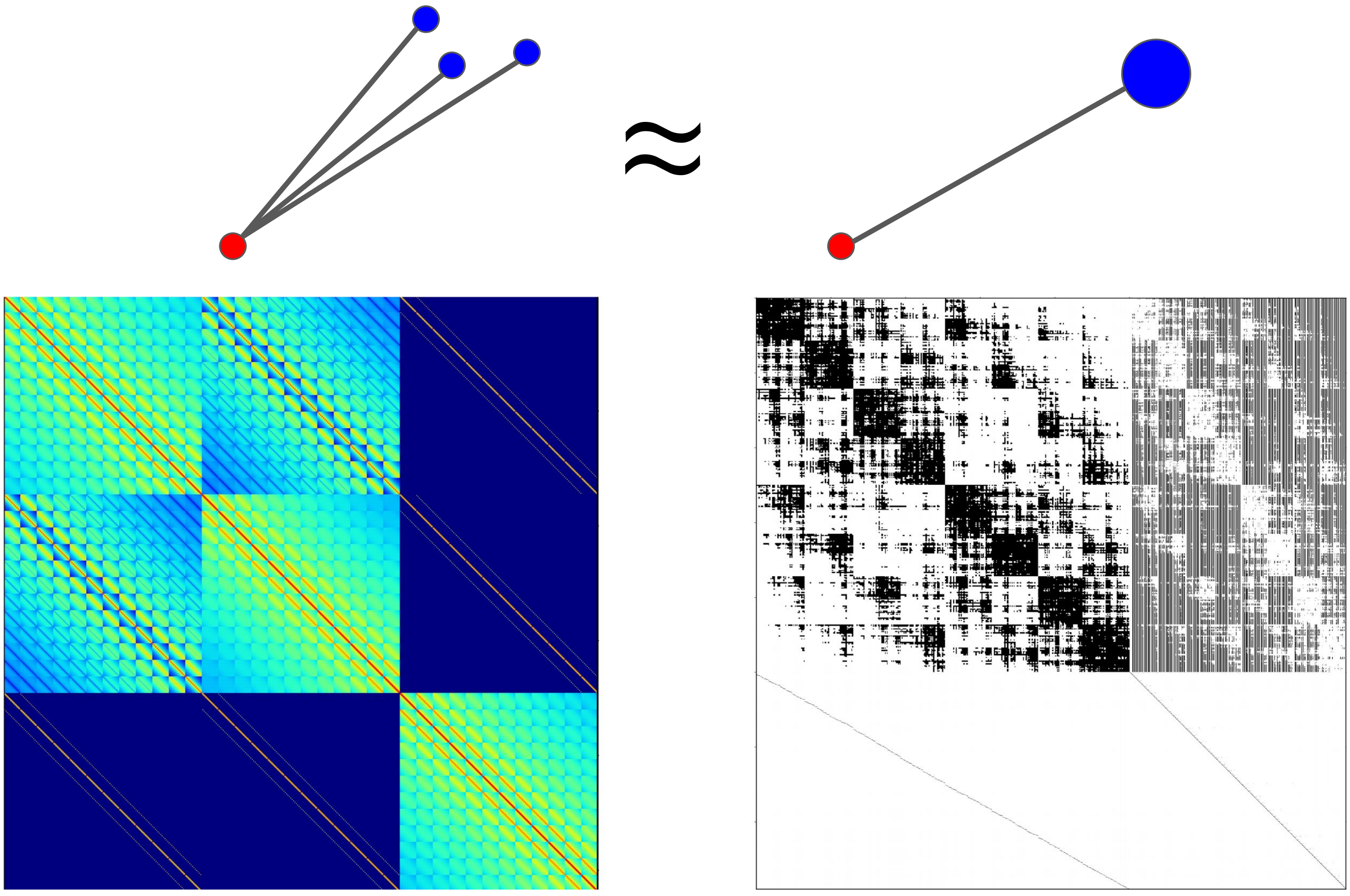
Two types of Green's function interaction integrals

Near-field



Far-field

Far-field: Dense BEM matrices are slow!
→ Fast multipole method



The near-field challenge:

$\longrightarrow$ any Green's function

$\longrightarrow$ any basis

$\longrightarrow$ high accuracy

$\longrightarrow$ black box to the user

Very difficult while remaining **fast** unless...

The near-field challenge:

$\longrightarrow$ any Green's function

$\longrightarrow$ any basis

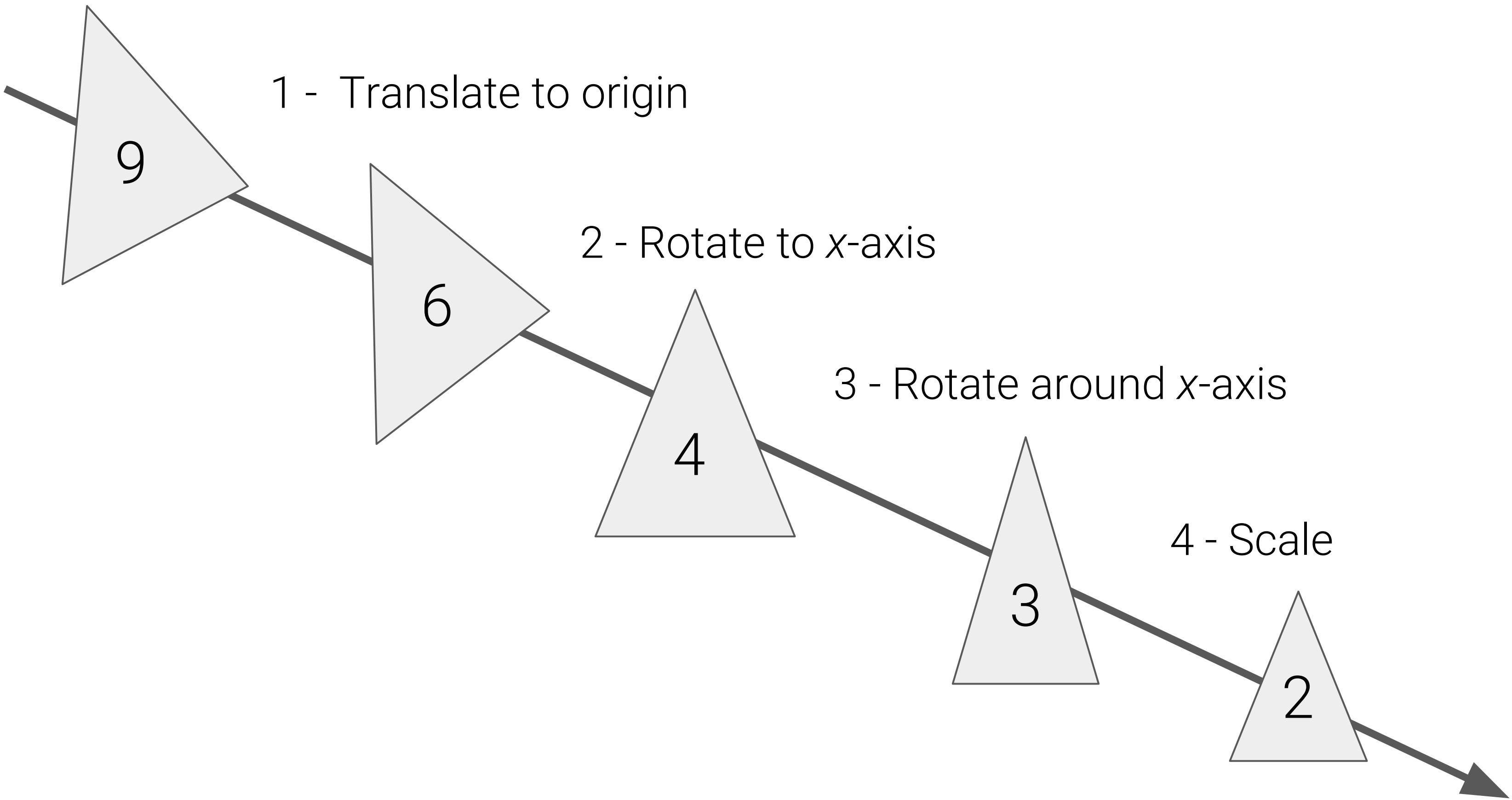
$\longrightarrow$ high accuracy

$\longrightarrow$ black box to the user

Very difficult while remaining **fast** unless...

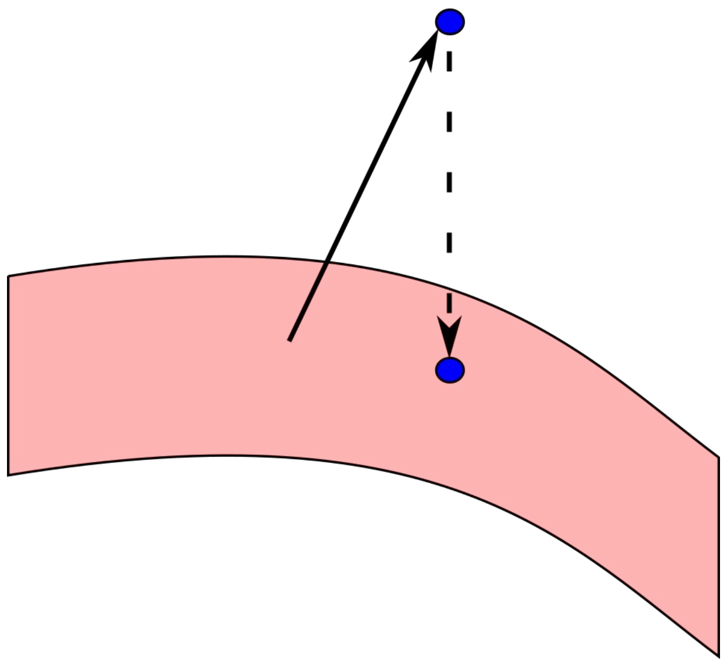
$\boxed{\text{Precompute}}$

Reduce the set of possible near-field integrals



Default: 9 geometric parameters and 2 material properties
Reduced: 2 geometric parameters and 1 material property

Avoid the singularity by taking a numerical limit…



and brute-force it with adaptive quadrature!

Tectosaur:
An efficient and flexible BEM for earthquake science

Near-field:

⟶ lookup tables built by brute-force

⟶ run-time lookup is very fast

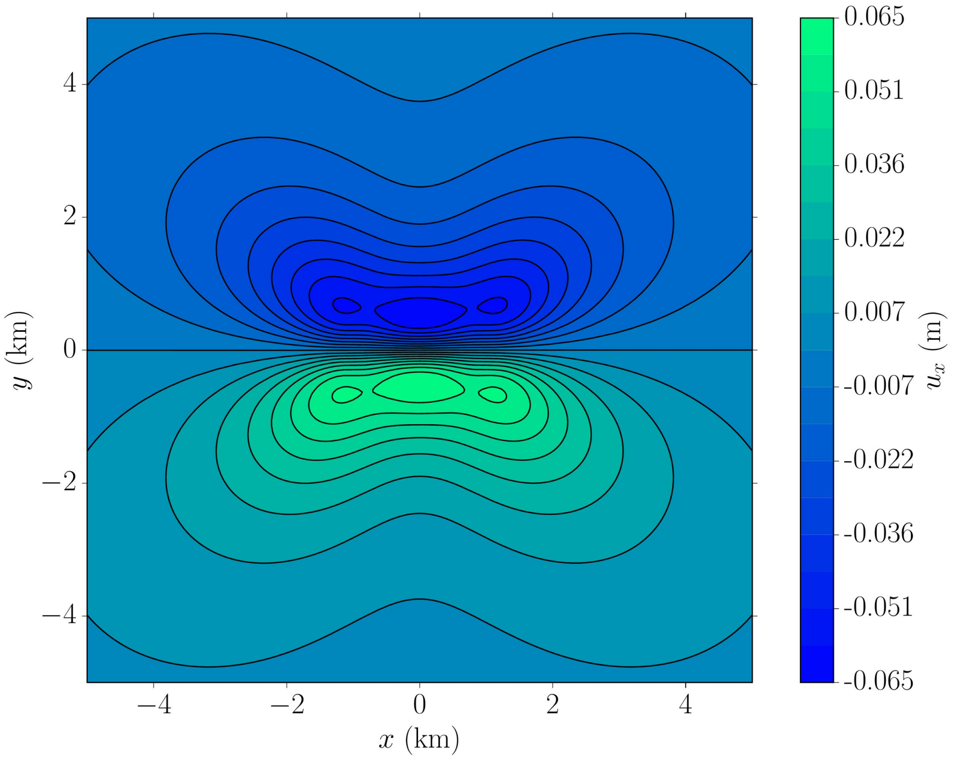
⟶ highly general and **black-box**

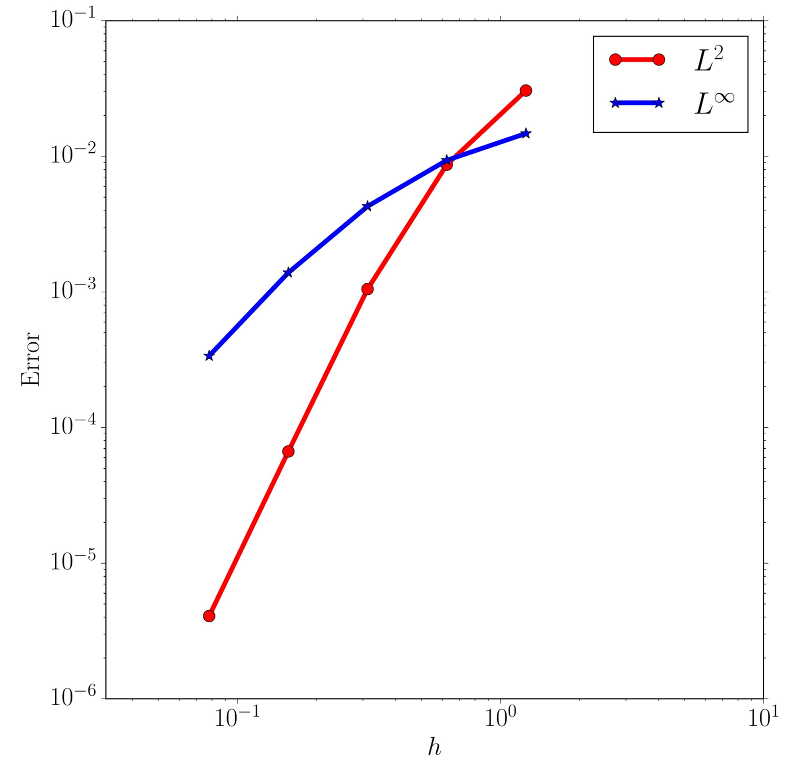
Far-field

⟶ fast multipole method enables **millions** of elements

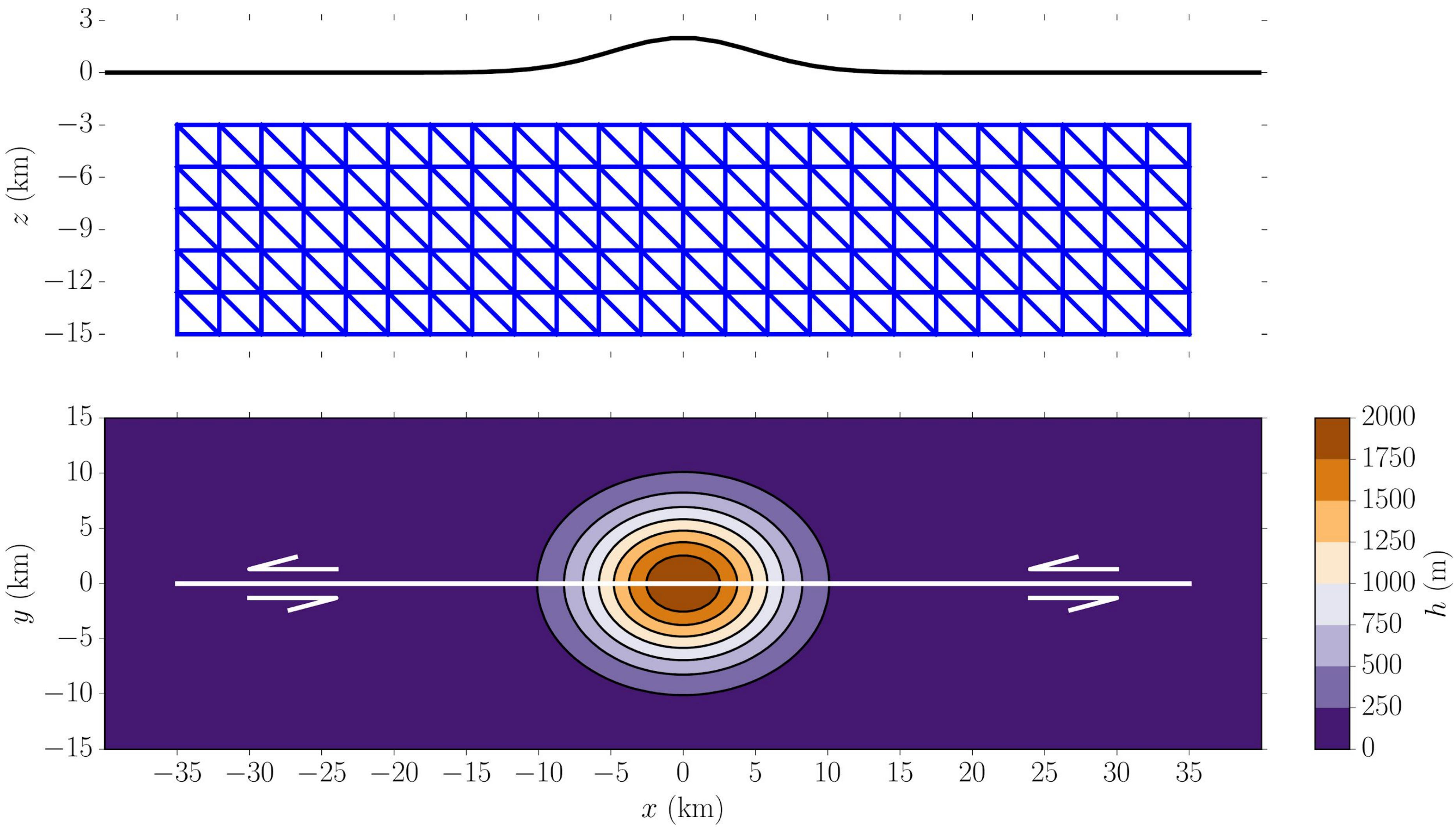
Parallel and GPU accelerated

A quick check: Can we replicate Okada?
(with full space Green's functions!)

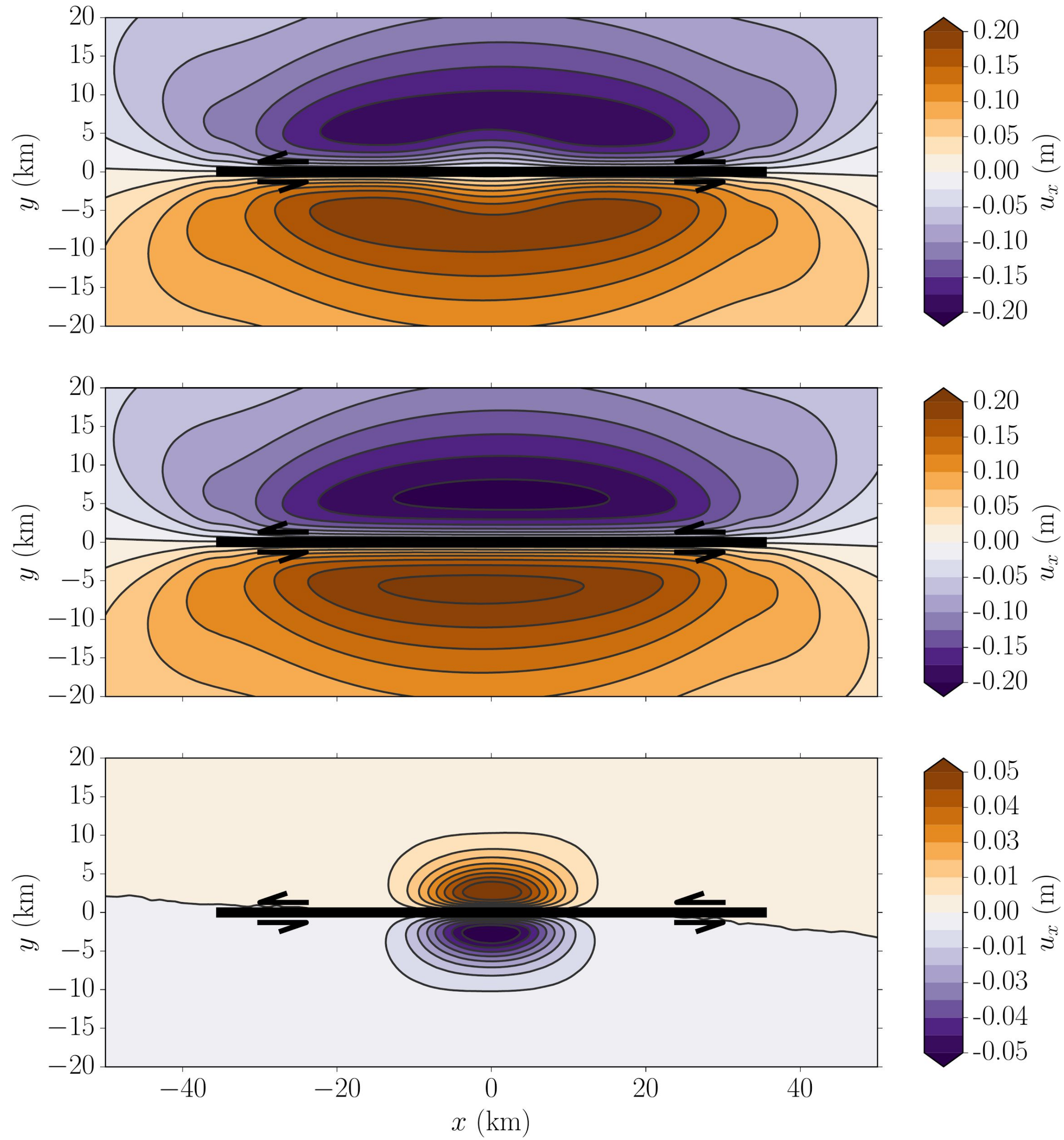




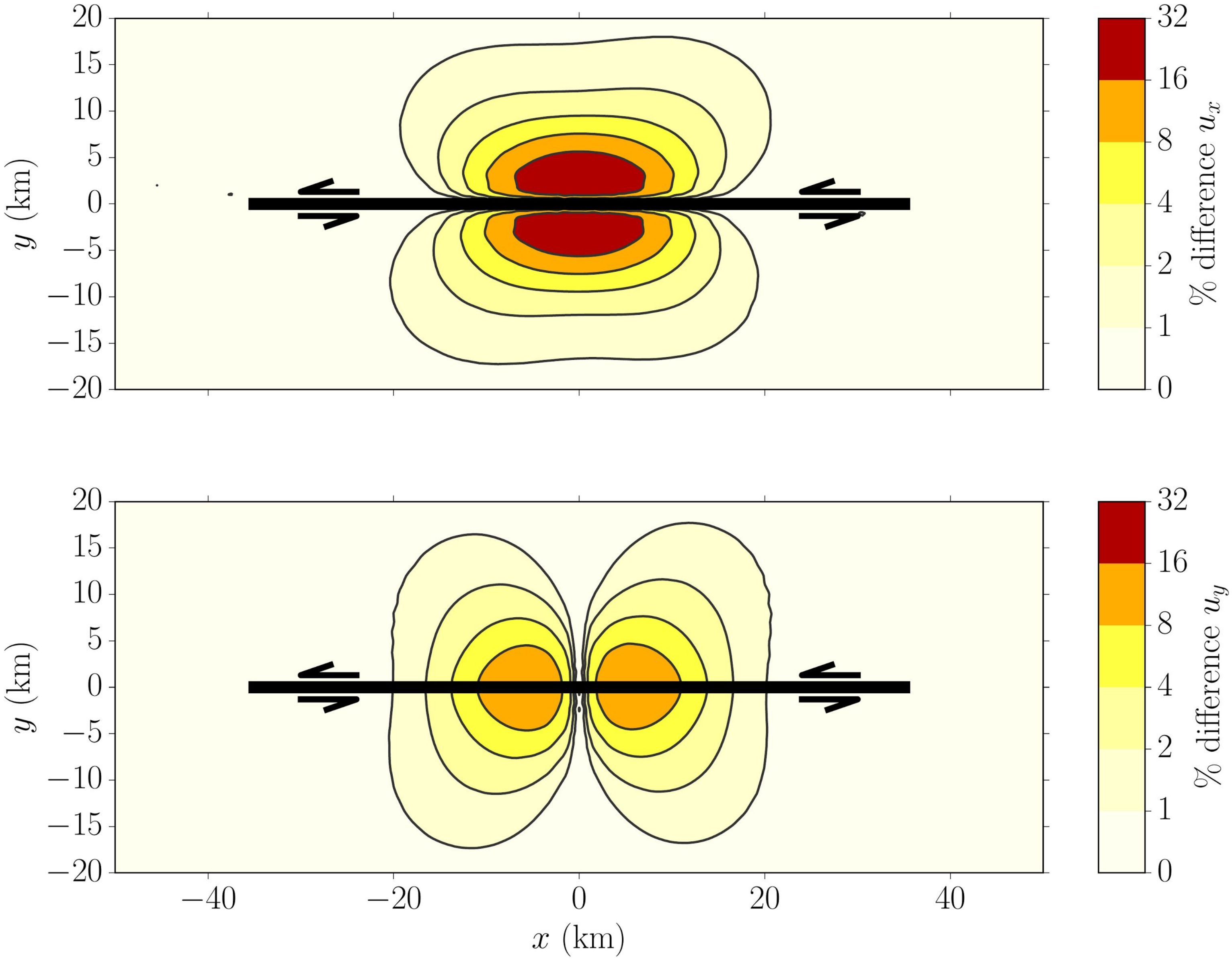
How does topography affect slip inversion?



$$U_{hill} = U_{flat} + U_{diff}$$



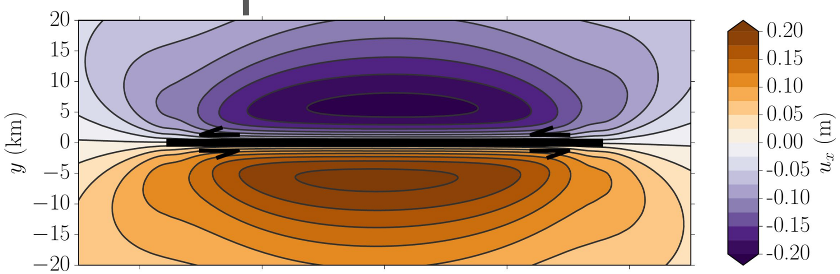
Differences are concentrated near steep areas

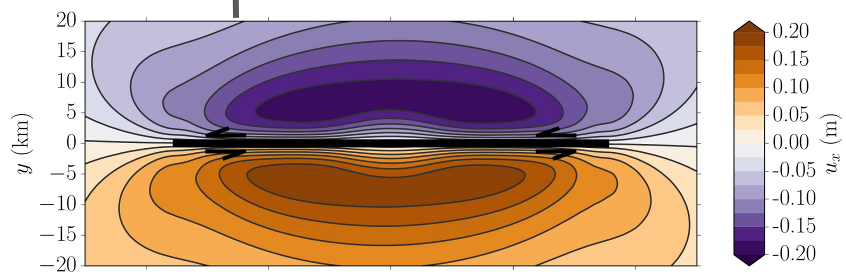


Invert

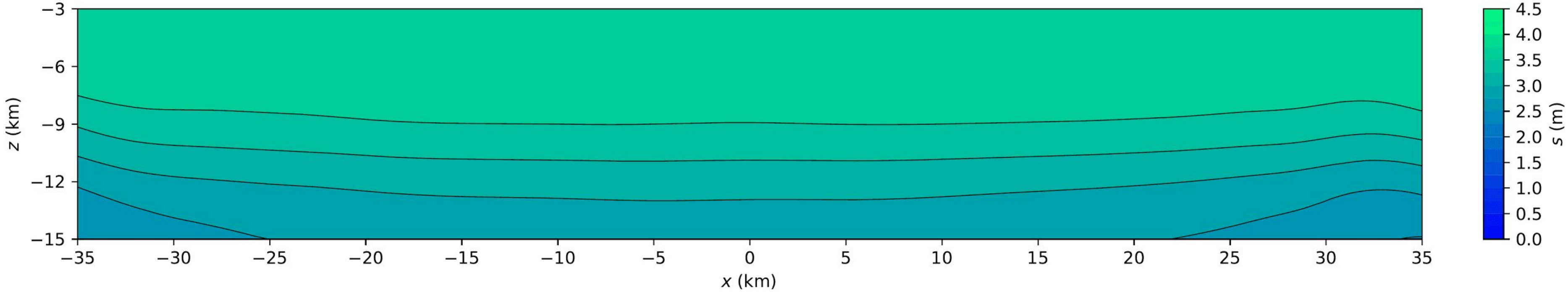
3.5 m uniform
strike-slip motion

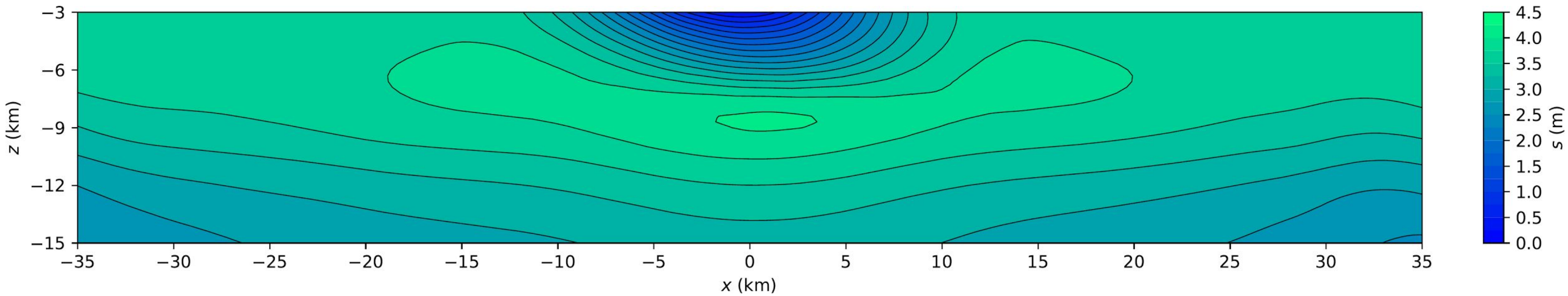
$$G_{flat} m_{flat} = G_{topo} m_{topo}$$



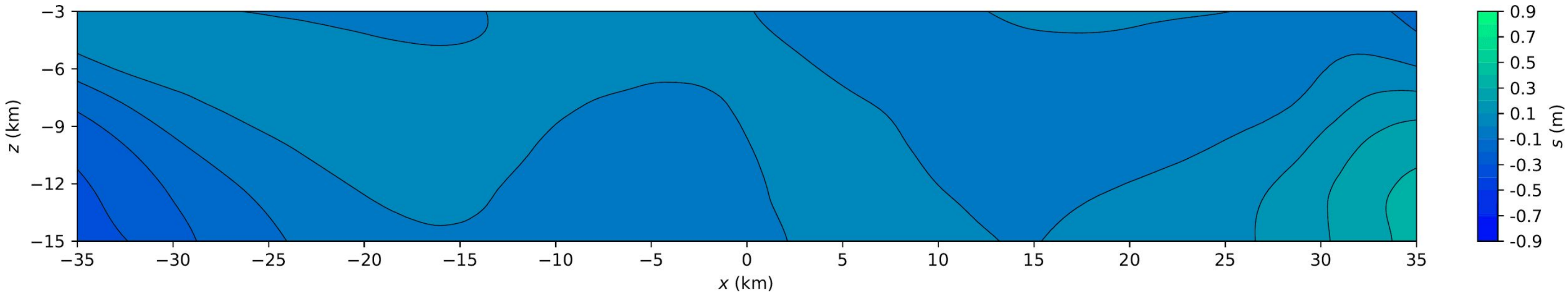


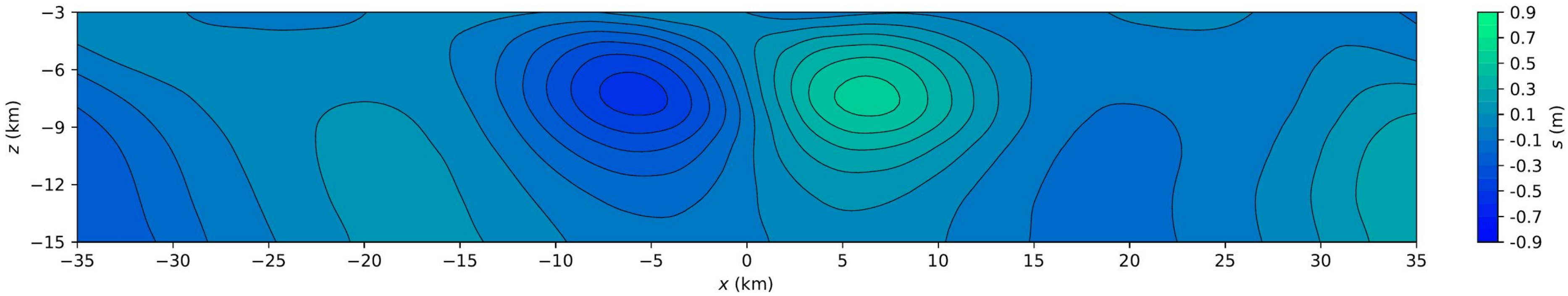
Ignoring topography → shallow slip-deficit



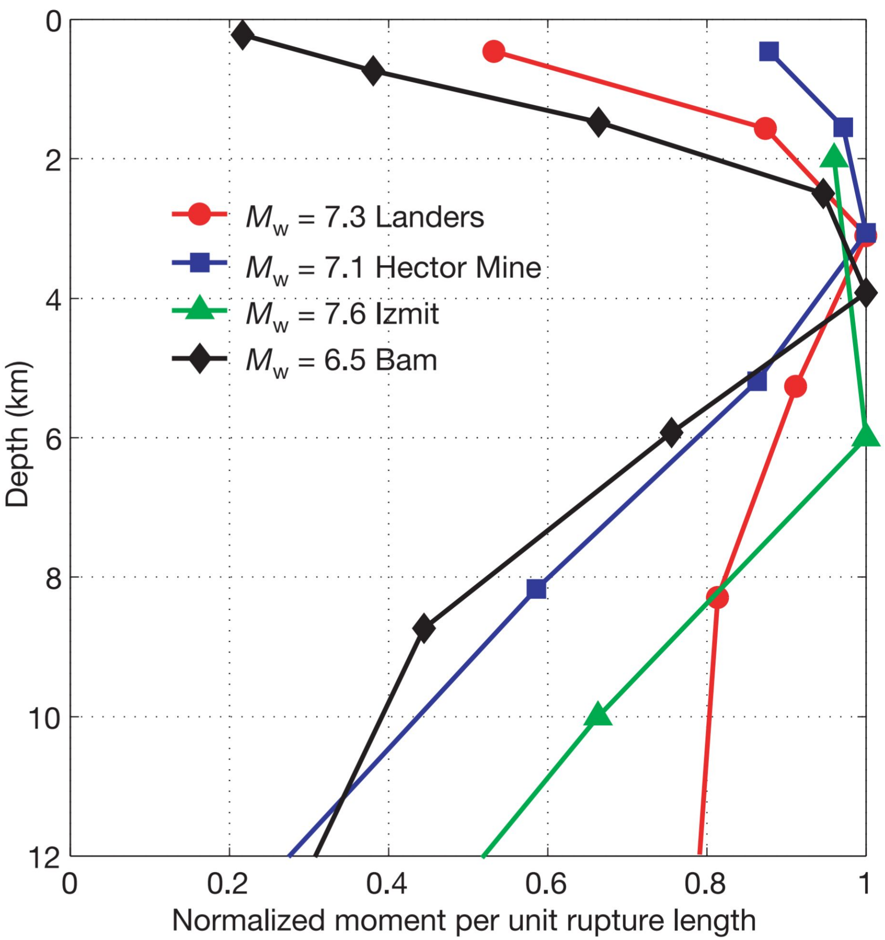


Ignoring topography → spurious dip-slip

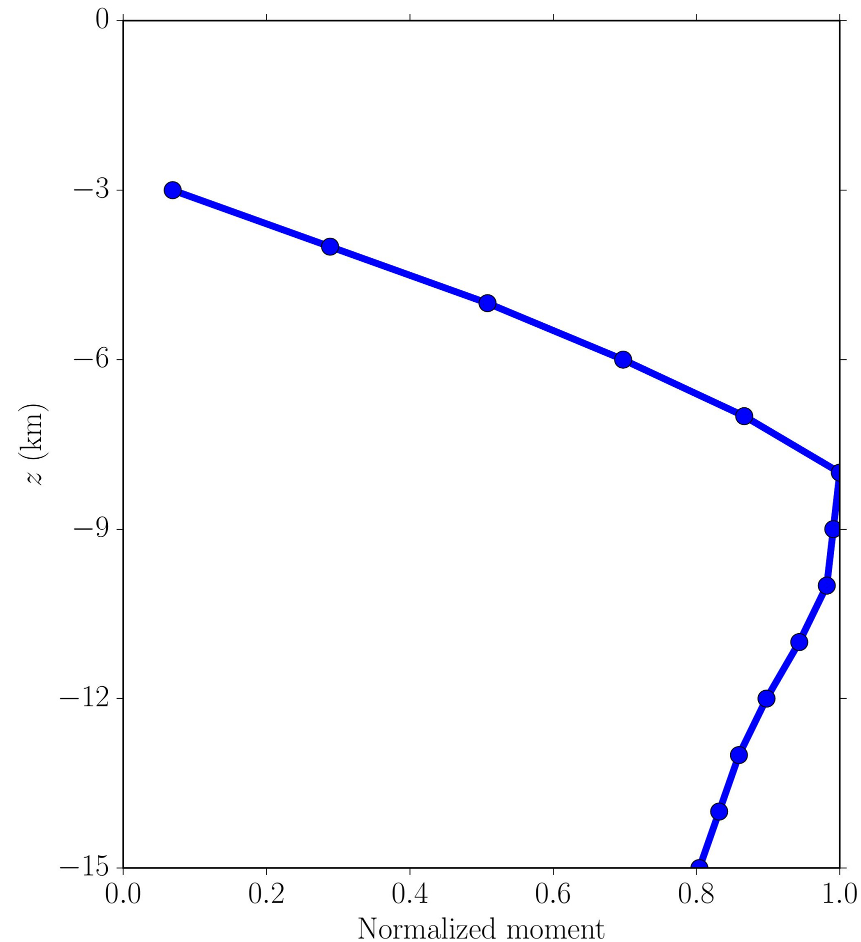




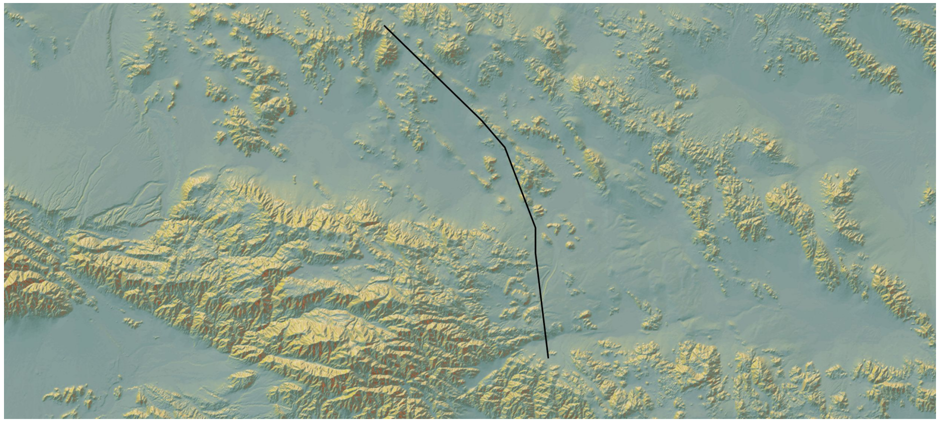
Shallow slip-deficit, deep slip-excess



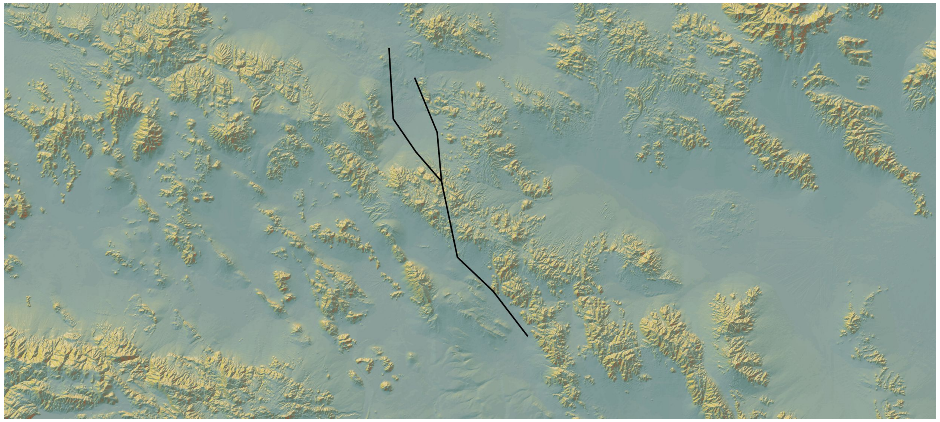
Fialko et al. 2005



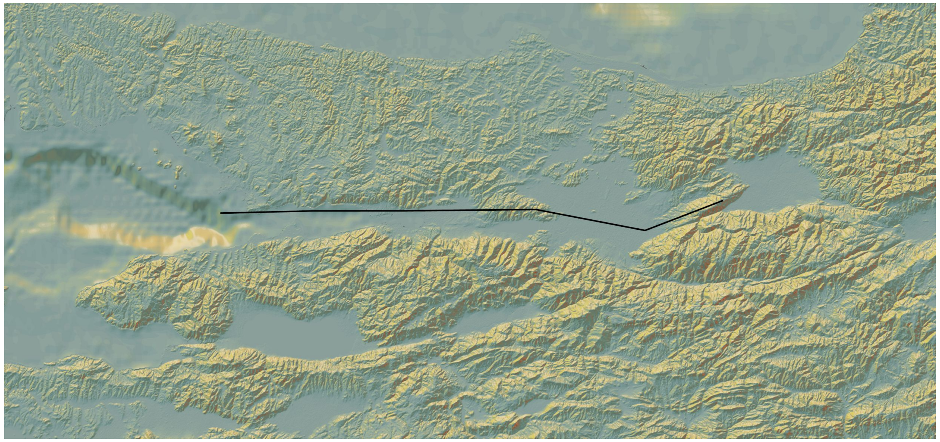
Landers 1992



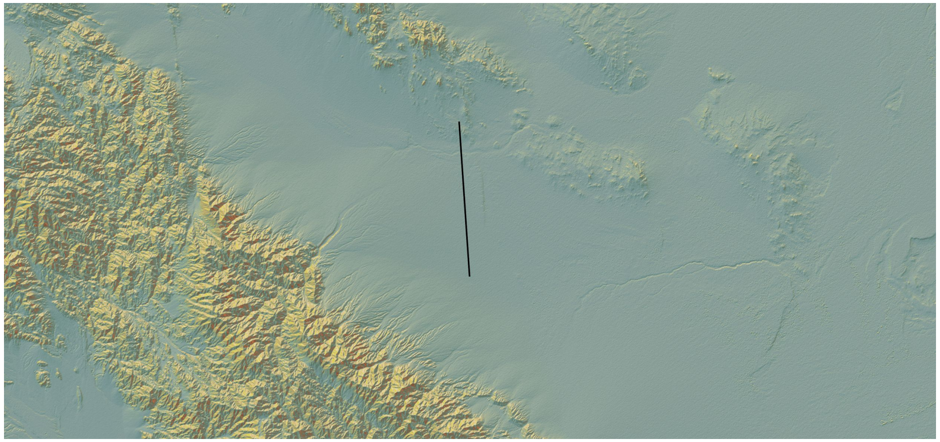
Hector Mine 1999



Izmit 1999



Bam 2003



How to explain shallow slip-deficits?

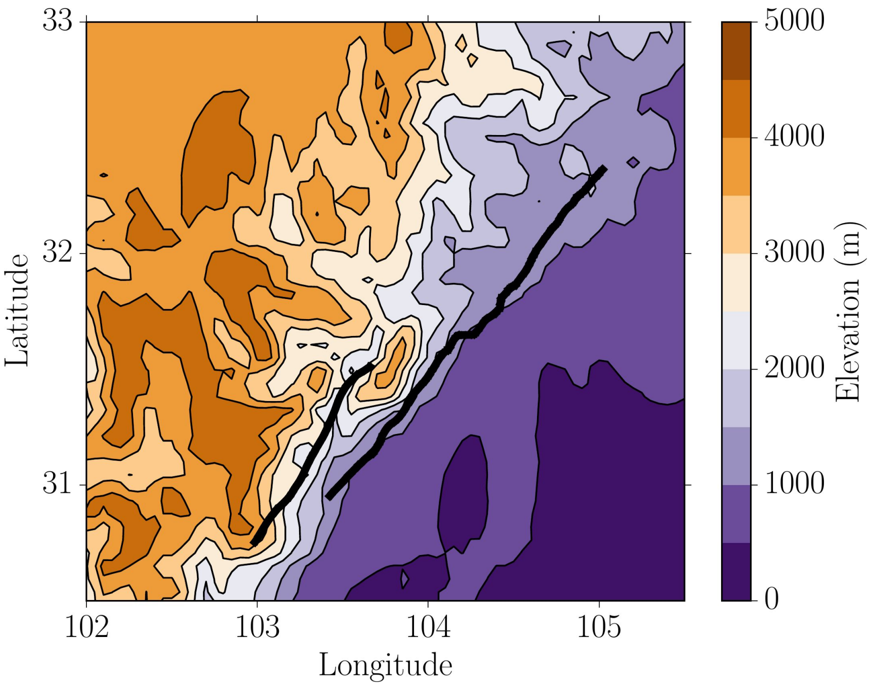
Inelastic near-surface deformation?

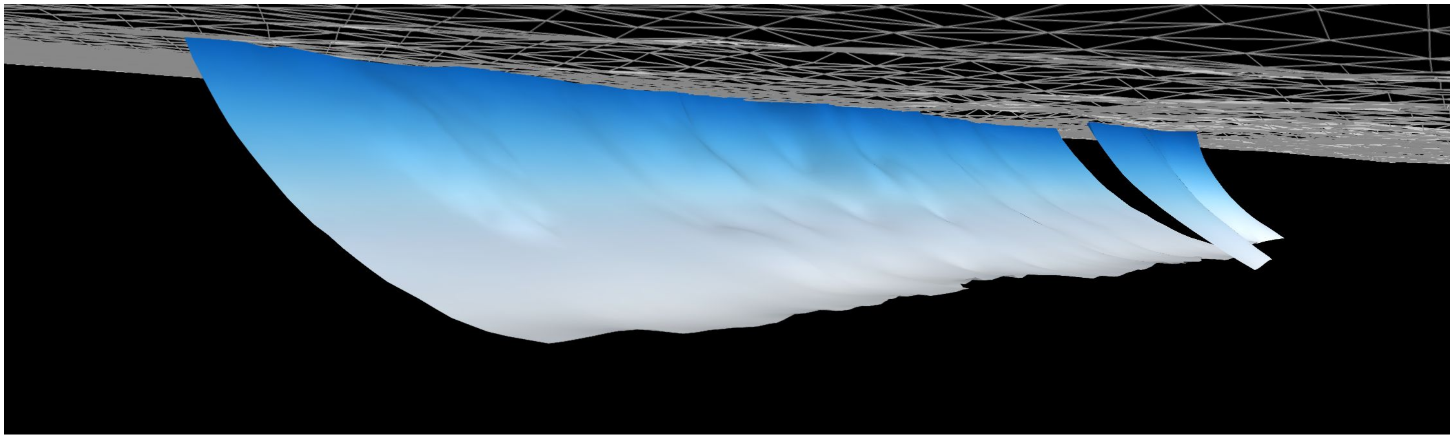
Interseismic/postseismic near-surface creep?

An artefact of ignoring free-surface effects of topography

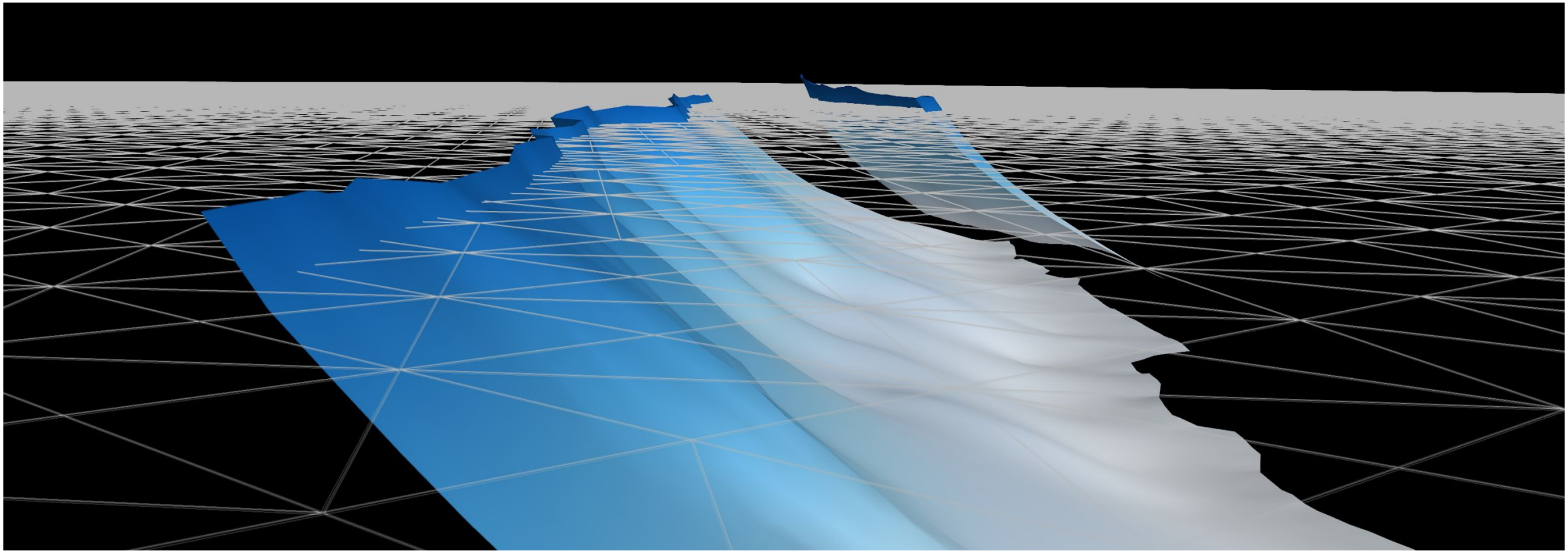
A real-world example: Wenchuan

Very steep:
Rises 5 km in 30 km





A difficult choice!



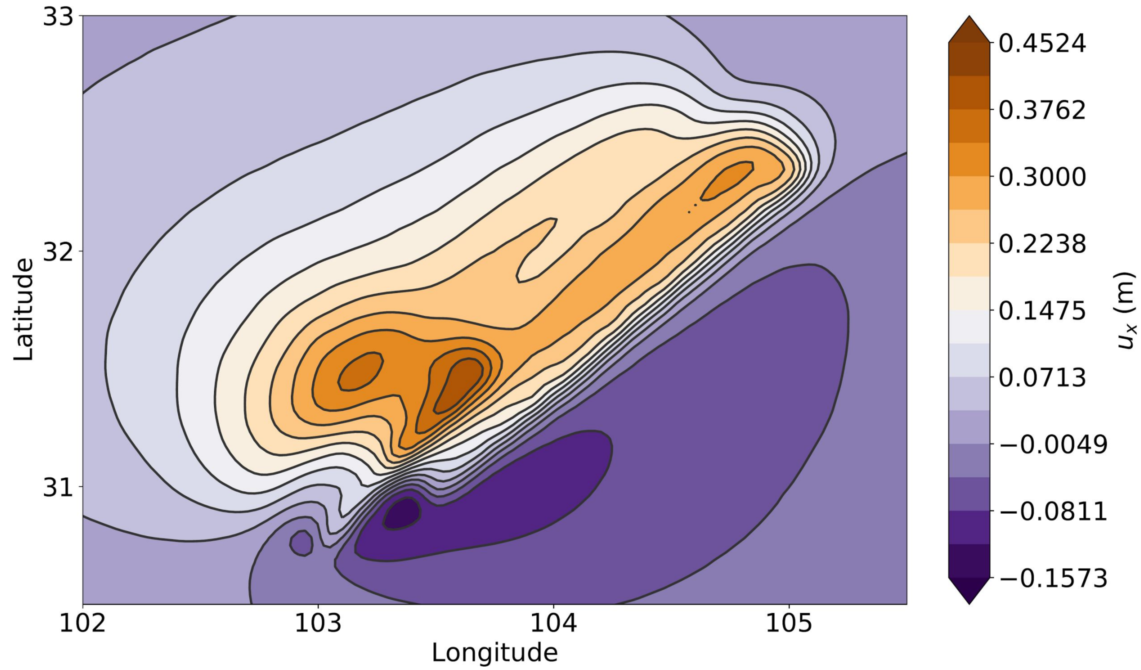
Do we:

use a free-surface that isn't at sea-level?

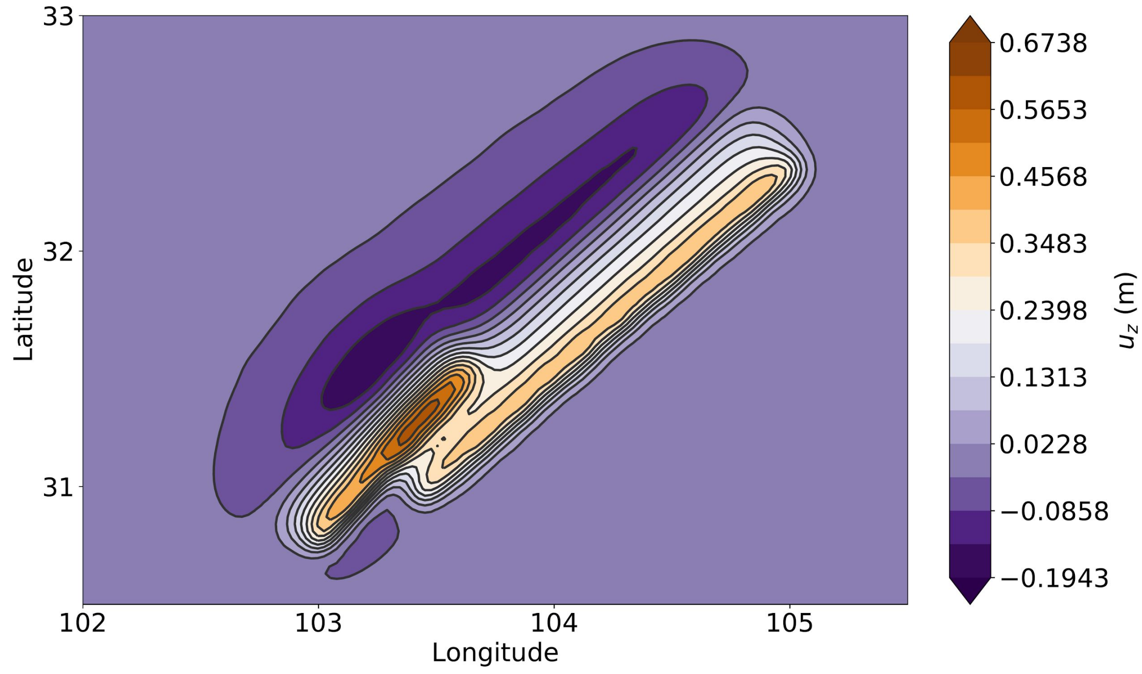
change the fault geometry so it lies below sea-level?

Forward model displacements (1 m of dip-slip)

U_X

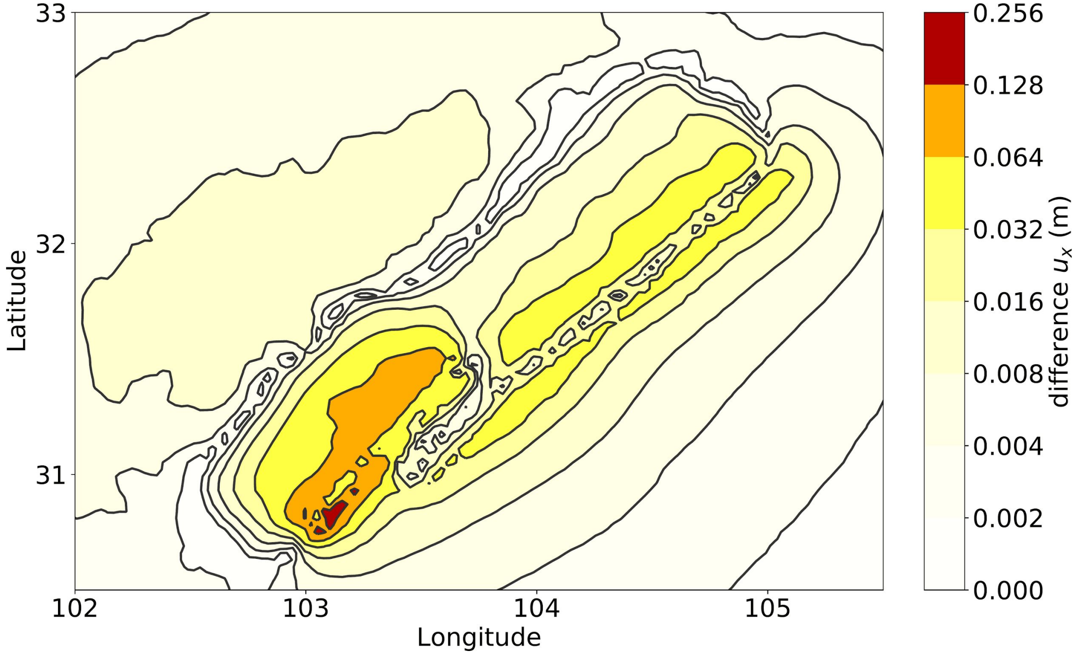


U_Z

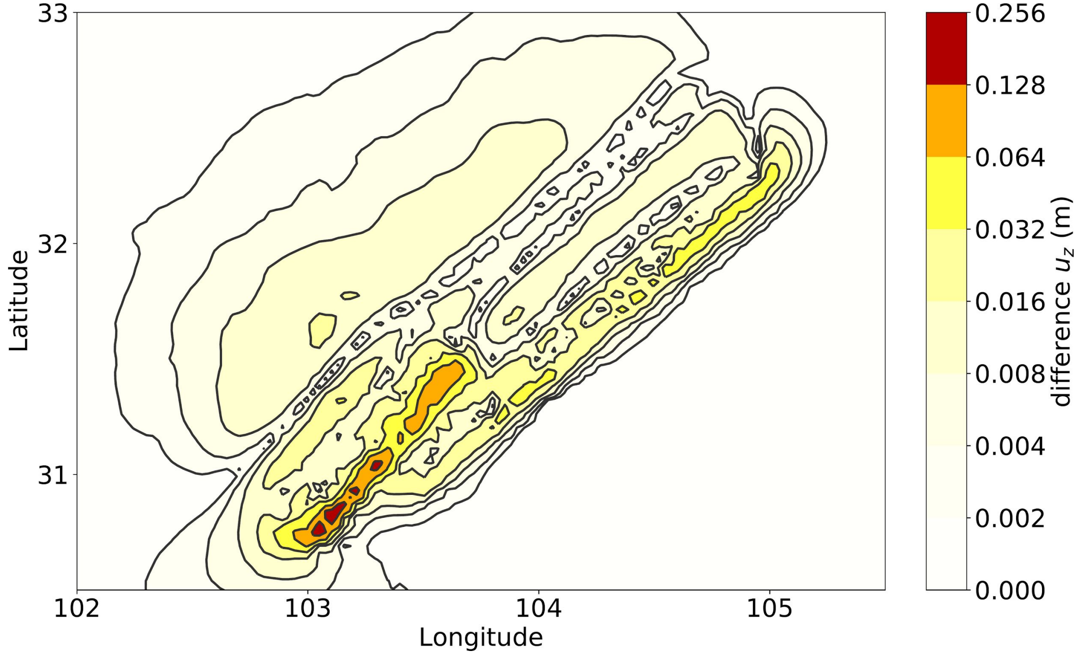


Differences are concentrated in the steep hanging wall

ΔU_X



ΔU_Z

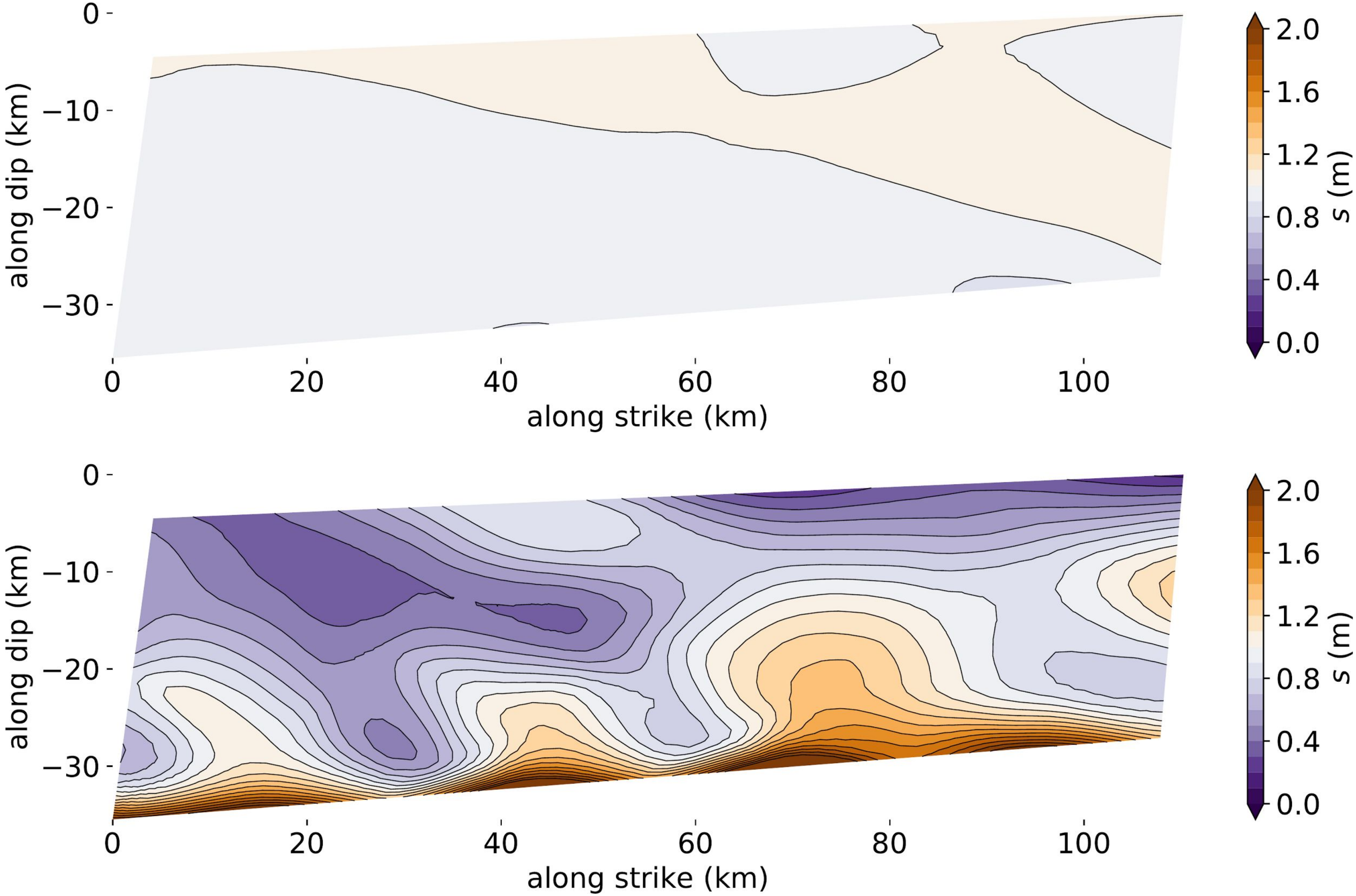


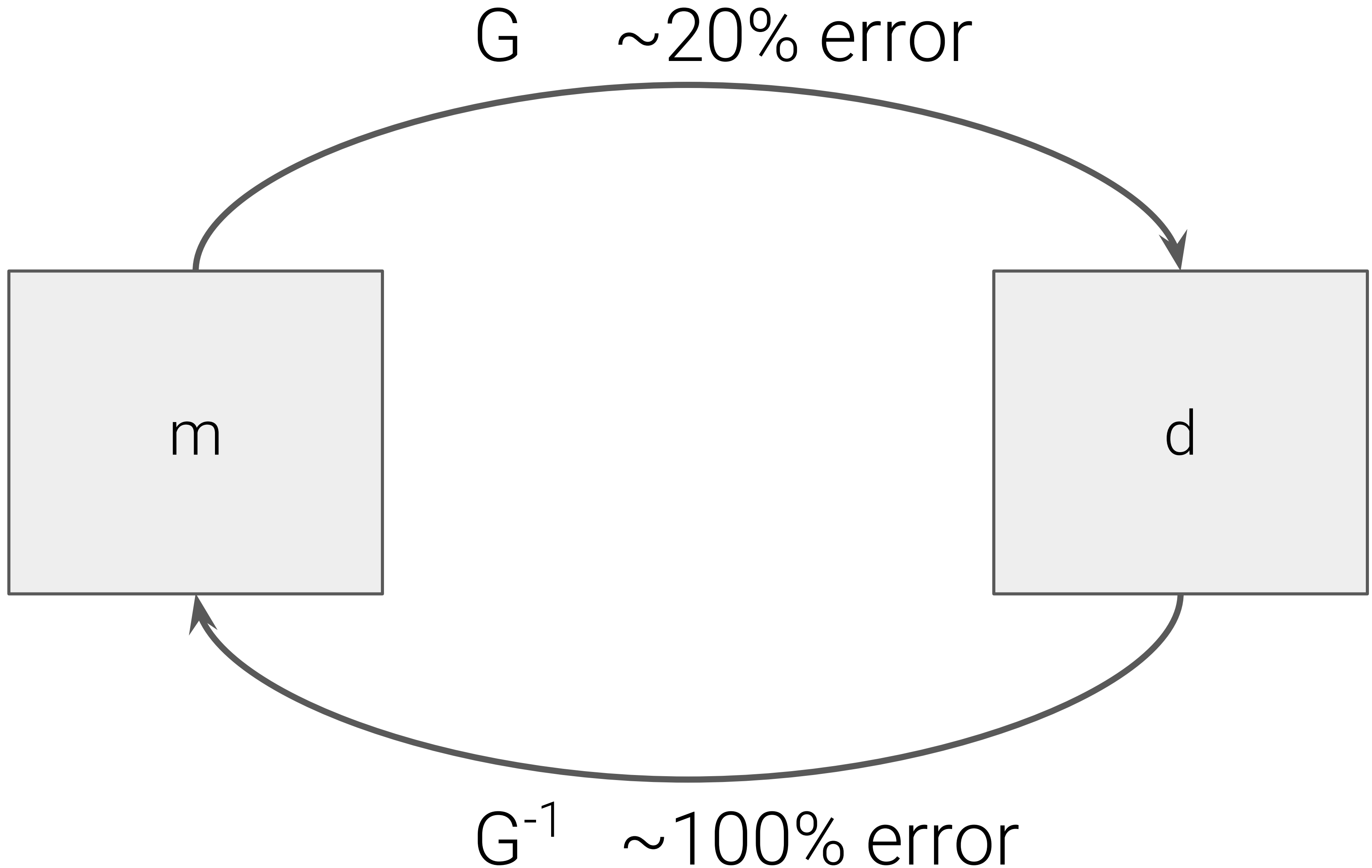
Invert

1.0 m uniform
dip-slip motion

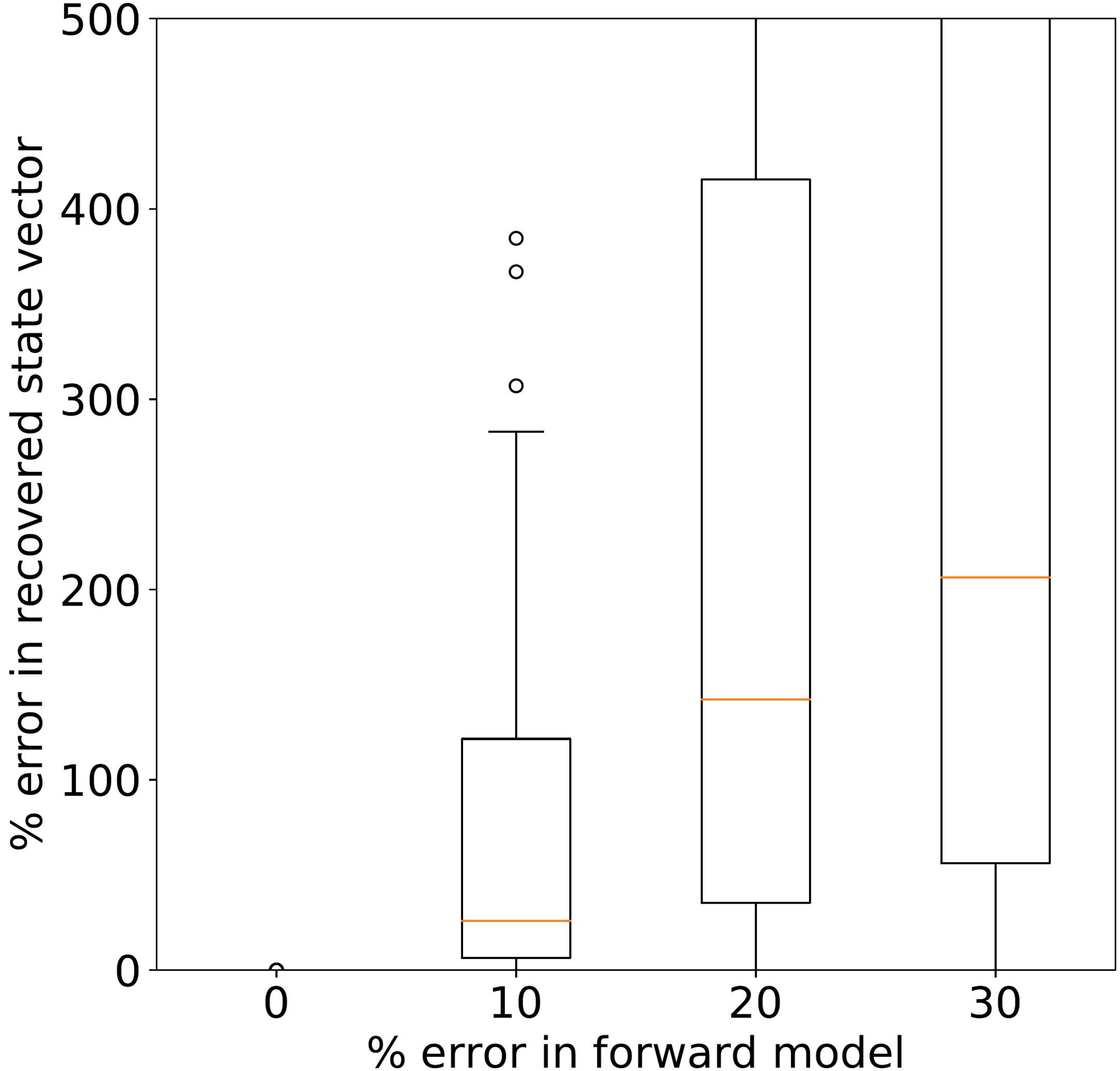
$$G_{flat} m_{flat} = G_{topo} m_{topo}$$

Topography controls the entire slip distribution




G ~20% error
m
d
G-1 ~100% error

Inversion amplifies forward model errors



Tectosaur: a new tool for high-fidelity fault modeling

Now:

topography
earth curvature
material contrasts

millions of elements
no volumetric meshing
rapid model iteration

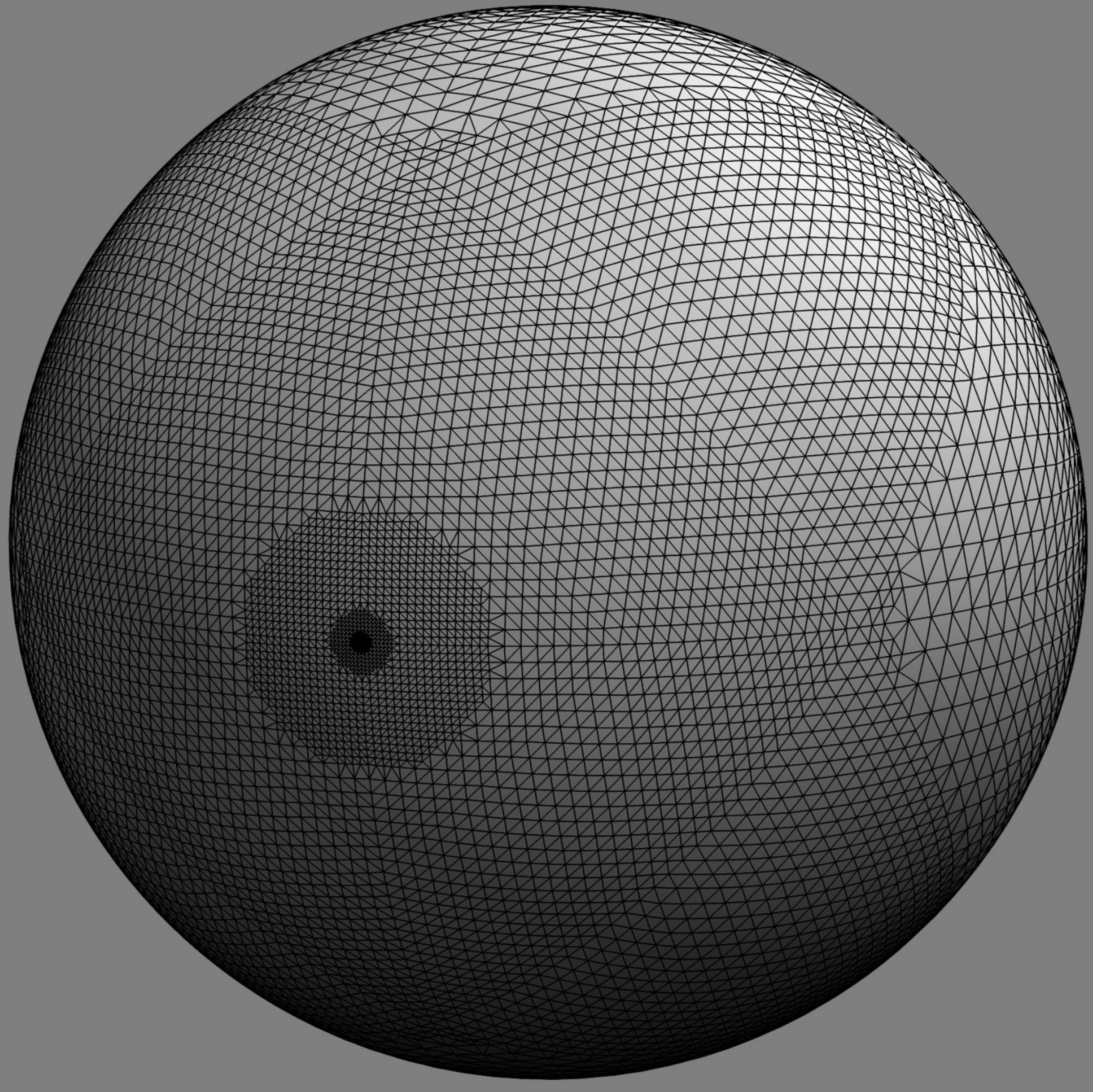
Ignoring topography creates inferred shallow slip-deficits

Small forward model effects can become huge inversion effects

Boundary element methods are not as limited as once thought

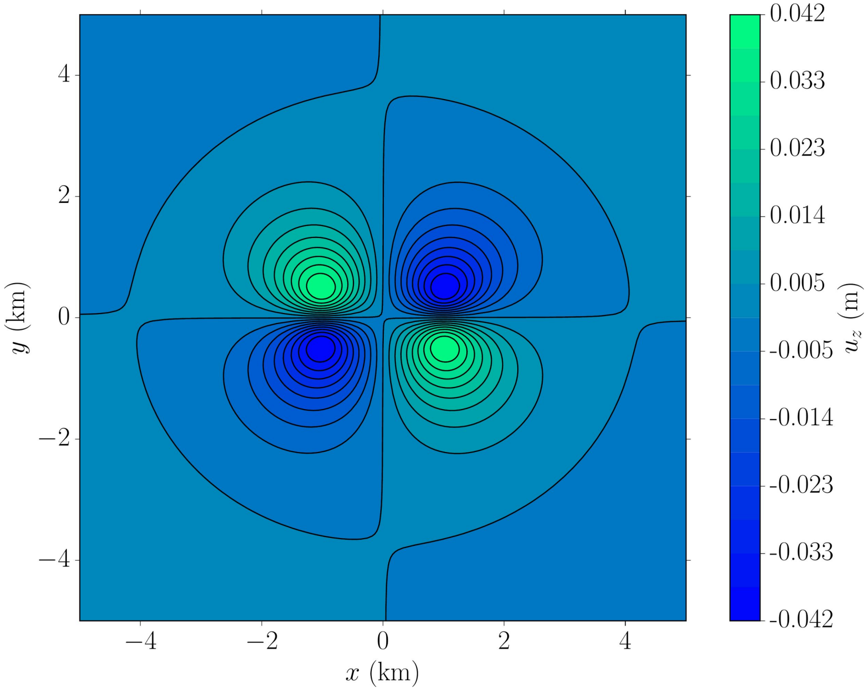
Future BEM:

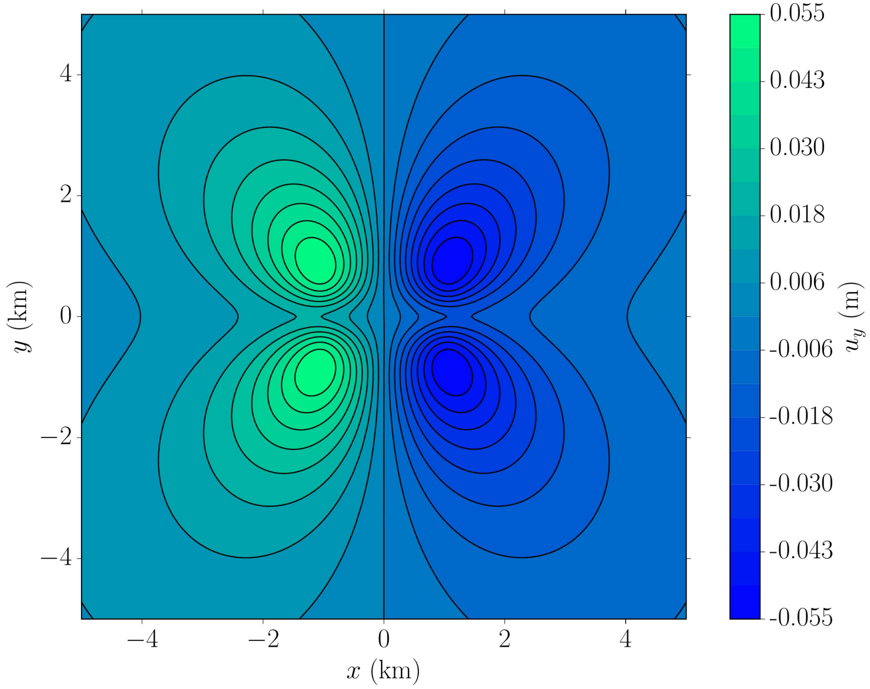
mesh-free nonlinearity? dynamic rupture/waves? YES



Extra slides

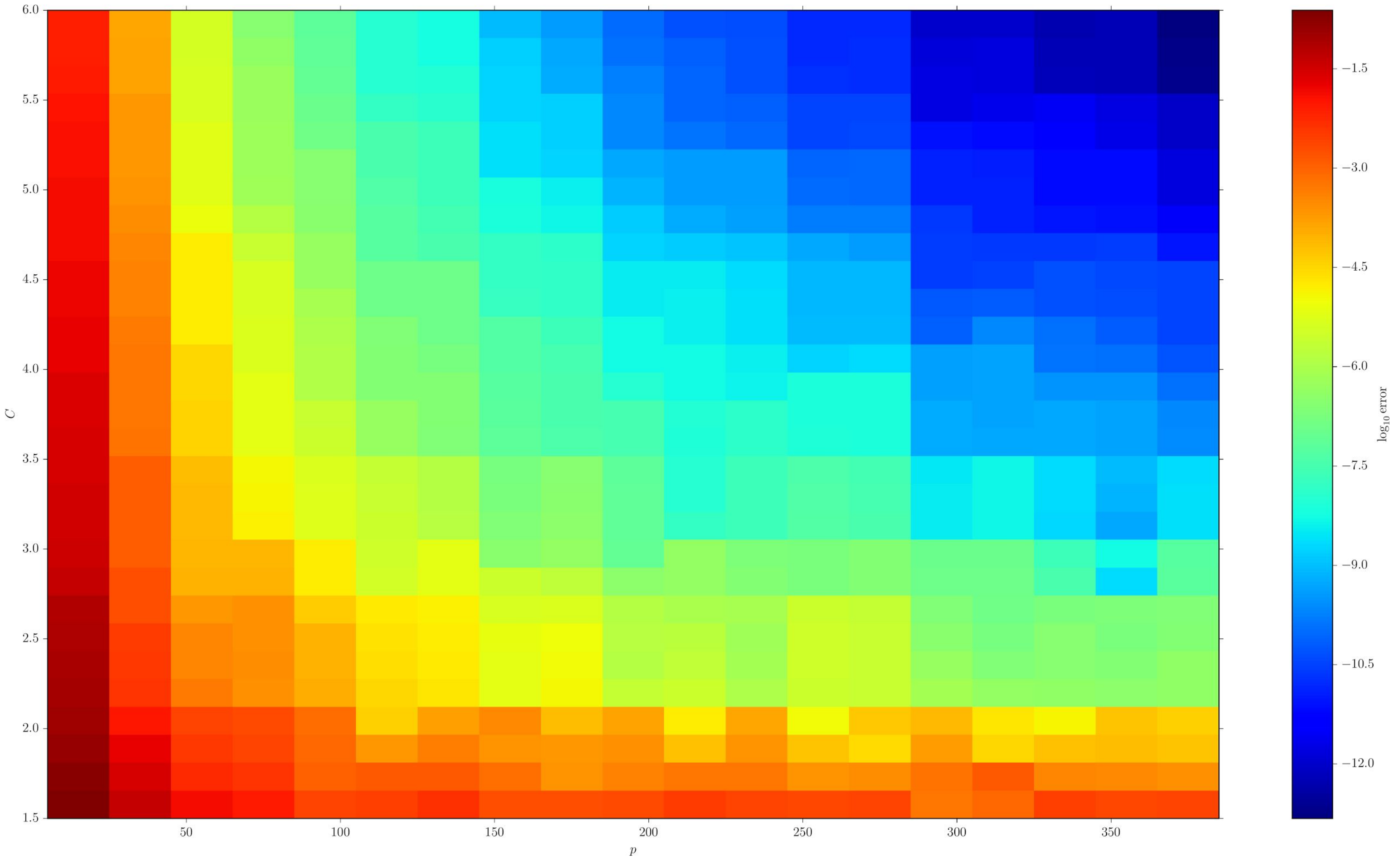
Okada, other dimensions



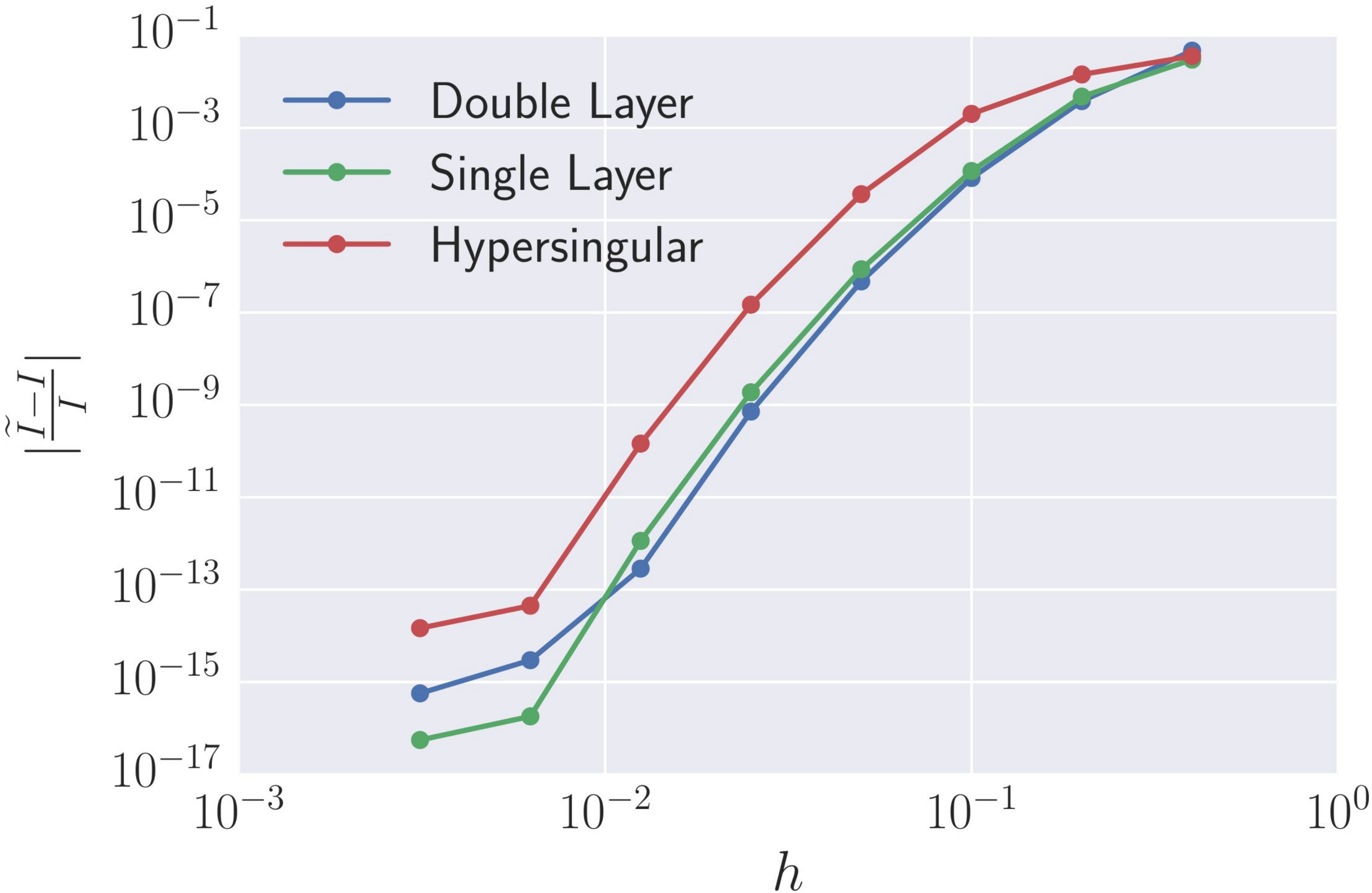


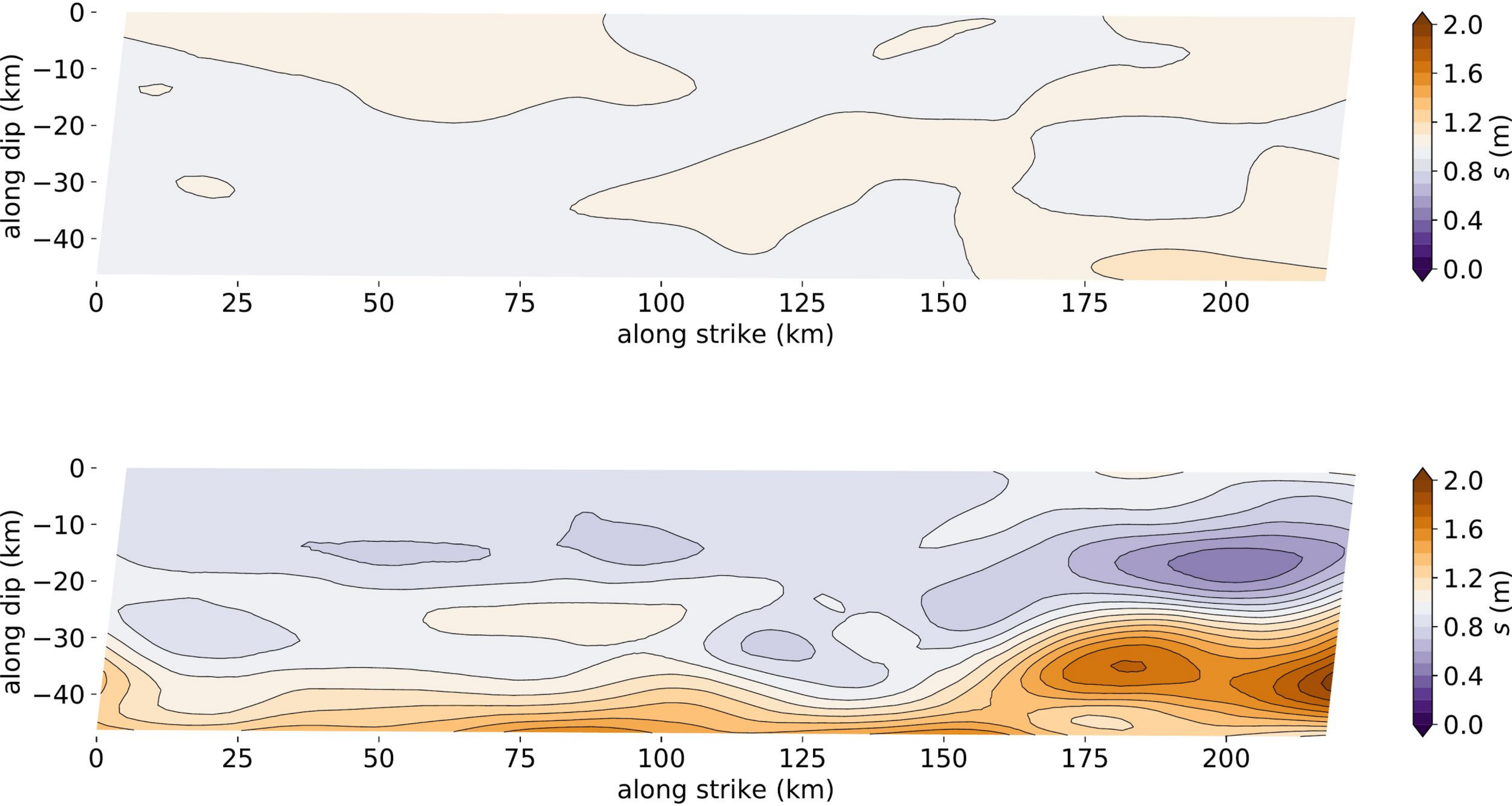
Also sphere compression works (whoa, we can do traction and displacement BCs?!)

FMM Log10 error



Richardson quadrature error for Laplace



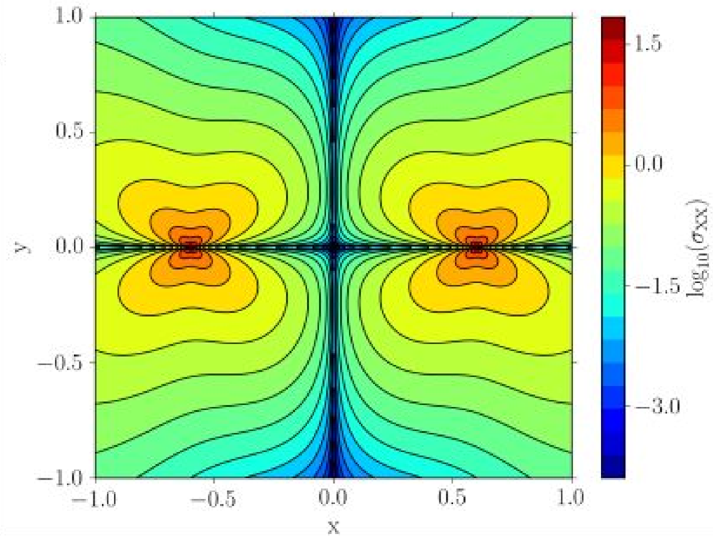


0
−10
−20
−30
−40
along dip (km)
0
25
50
75
100
125
150
175
200
along strike (km)
2.0
1.6
1.2
0.8
0.4
0.0
s (m)
0
−10
−20
−30
−40
along dip (km)
0
25
50
75
100
125
150
175
200
along strike (km)
2.0
1.6
1.2
0.8
0.4
0.0
s (m)

Old slides

Dislocation-based boundary element methods make these problems very hard

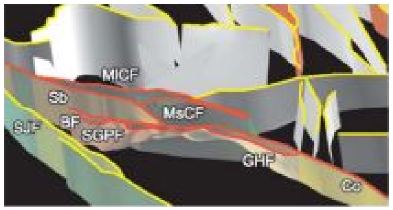
Rectangular dislocations (*Okada*, 1994) and triangular dislocations (*Meade*, 2007) are limited to **constant slip per element**.

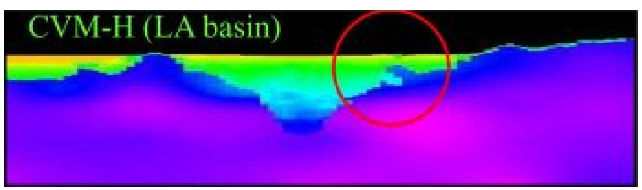


This leads to:

- Discontinuous displacement between elements
- Stress singularities at the edges of elements

Goal: earthquake cycle modeling and simulation in large geometrically complex fault systems





Plesch et al. 2007

Complex geometries

Rapid model iteration
(no volumetric meshing!)

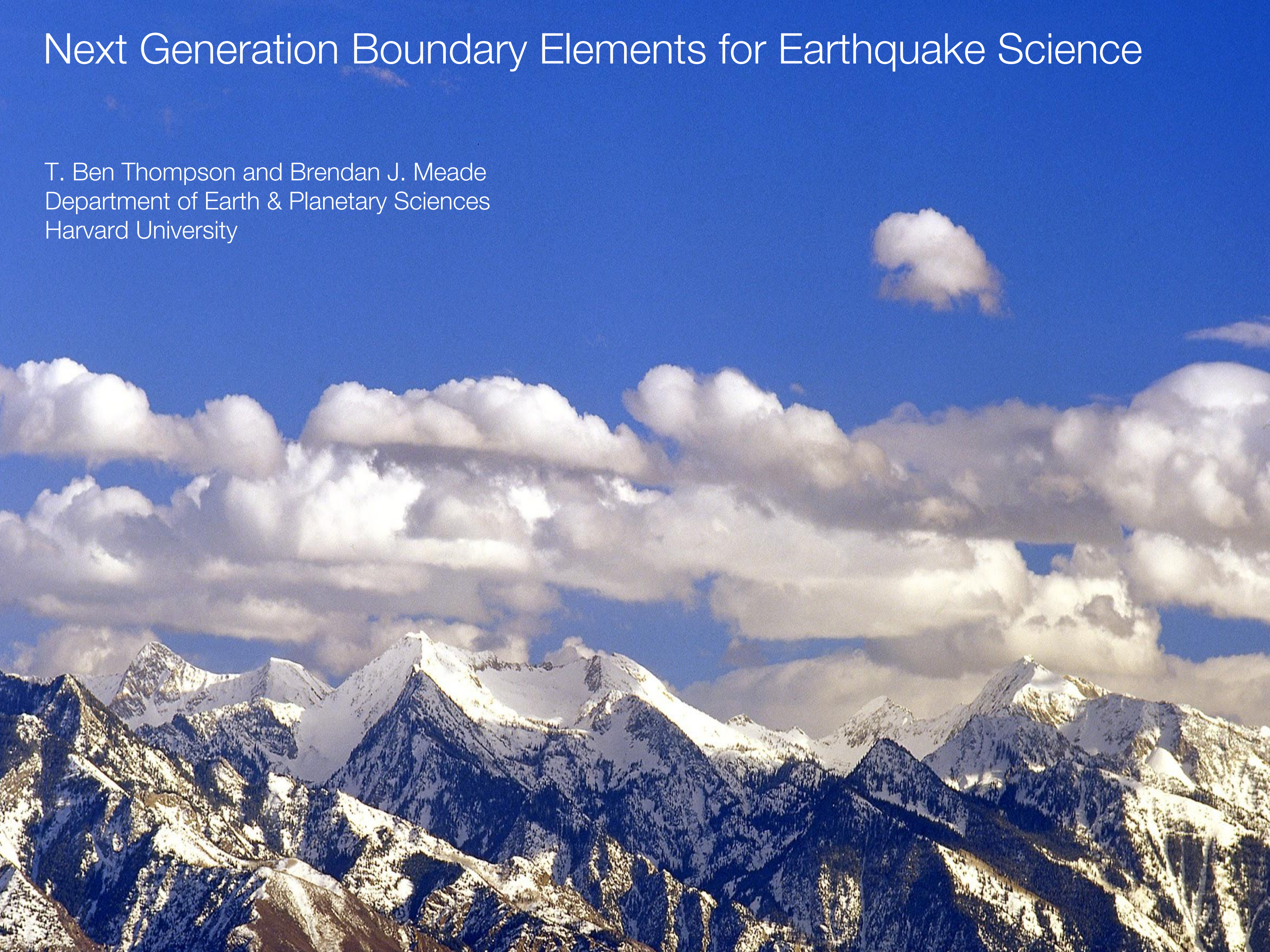
Gravitational stresses

Surface topography

Sharp material contrasts

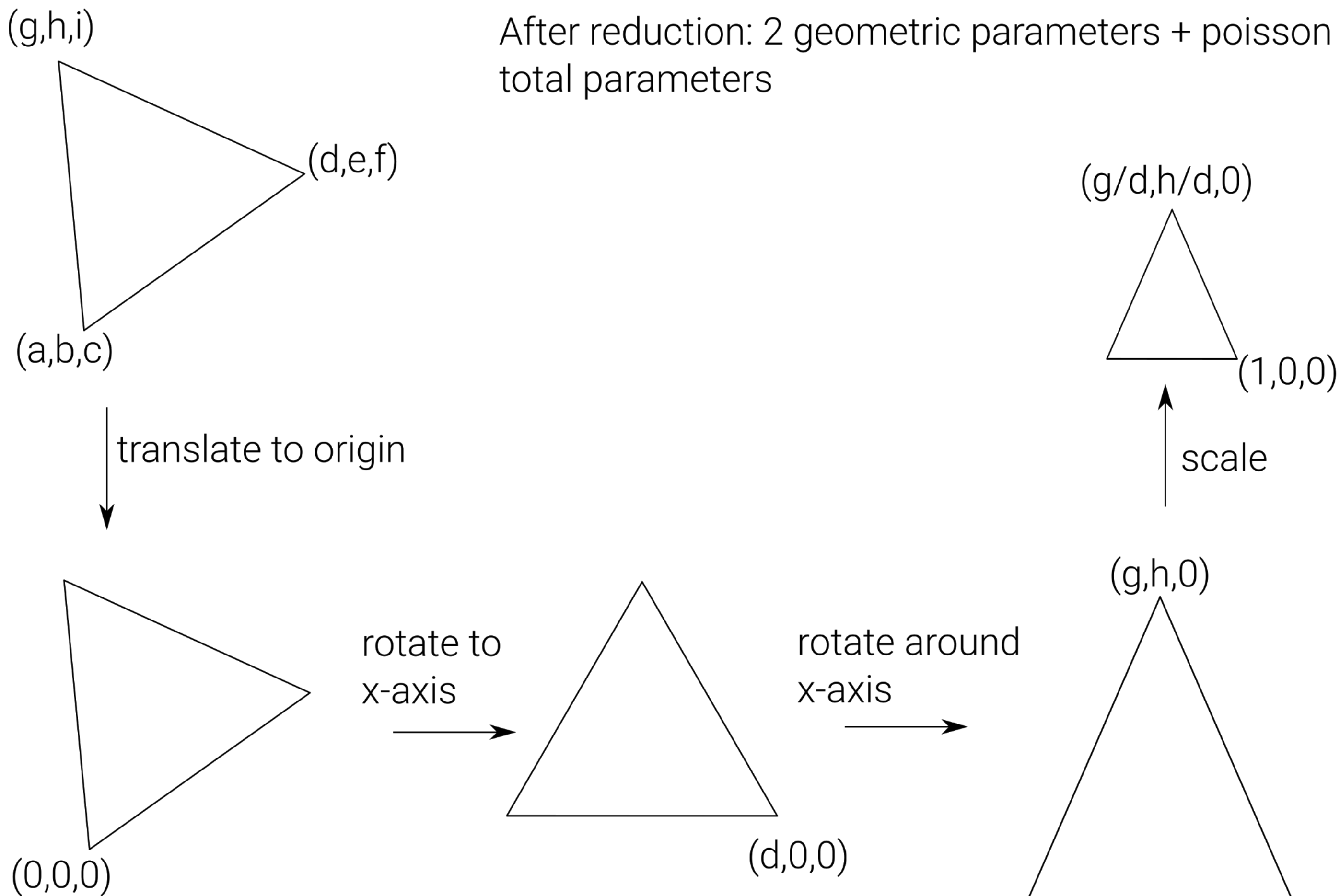
Next Generation Boundary Elements for Earthquake Science

T. Ben Thompson and Brendan J. Meade
Department of Earth & Planetary Sciences
Harvard University



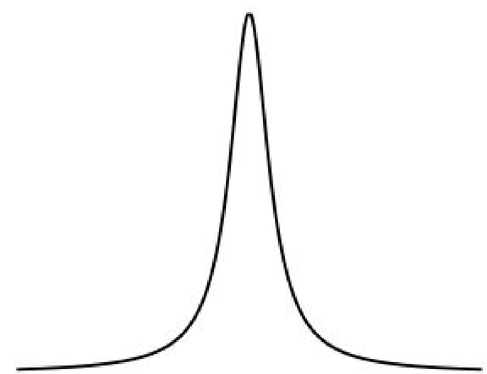
Initially: 9 geometric parameters + 2 material (shear modulus, poisson ratio)

After reduction: 2 geometric parameters + poisson ratio = 3 total parameters



Extrapolation converts one singular integral into several nearly singular integrals

Problem: Computing nearly singular integrals is expensive because these integrals are still very peaked (and 4D!)



Solution: An interpolation table for these integrals can be computed offline.

General Boundary Element Methods are Ideal

Instead of analytical dislocation theory, use numerically computed surface integrals

$$u_i(x) + \int_S U^*_{i,j}(x,y)t_j(y)dy = \int_S T^*_{i,j}(x,y)u_j(y)dy$$

$$U^*_{i,j}(\vec{x},\vec{y}) = \frac{1}{16\pi\mu(1-\nu)r}[(3-4\nu)\delta_{ij} + r_{,i}r_{,j}]$$

$$T^*_{i,j}(\vec{x},\vec{y}) = \frac{-1}{8\pi(1-\nu)r^2}\left[\{(1-2\nu)\delta_{ij} + 3r_{,k}r_{,j}\}\frac{\partial r}{\partial n} - (1-2\nu)\{n_j r_{,k} - n_k r_{,j}\}\right]$$

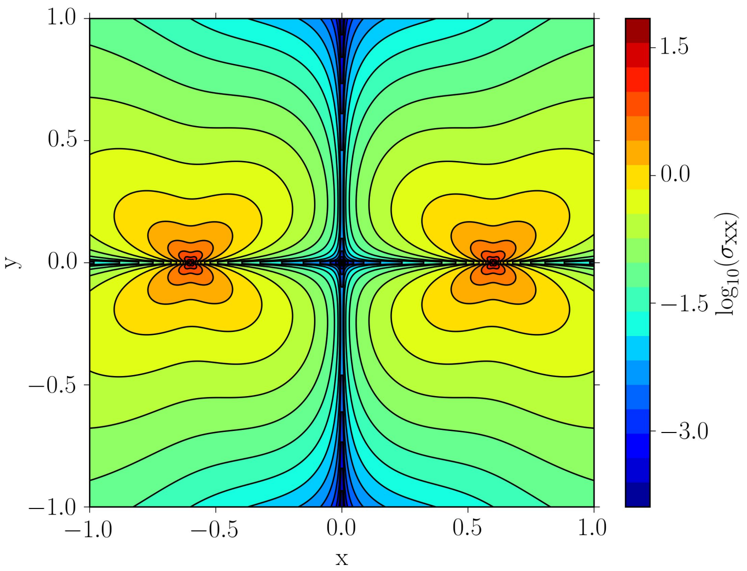
$$r = \|x-y\| \qquad r_{,i} = \frac{\partial r}{\partial x_i}$$

Linear basis functions allow continuous displacement distributions → No stress singularities!

Gravity, topography, and material contrasts can all be treated accurately.

Current boundary element methods are also limited

Constant slip Okada dislocations produce unphysical singular stresses



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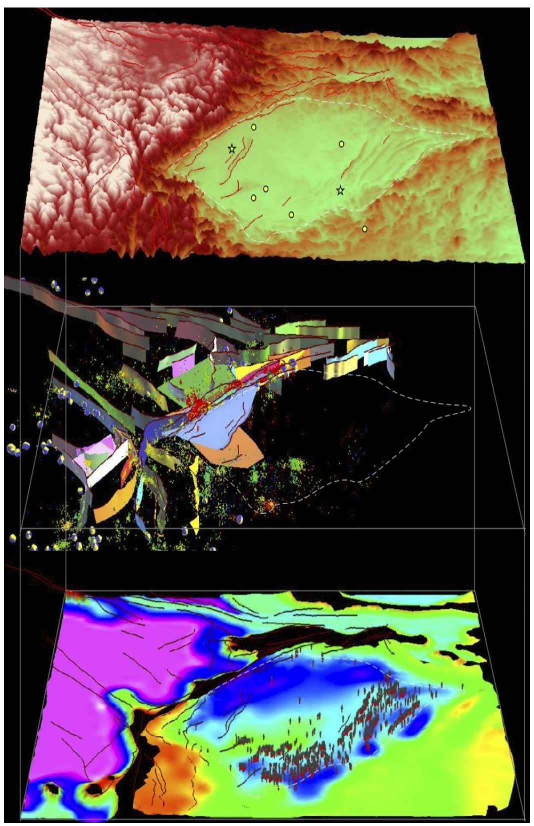

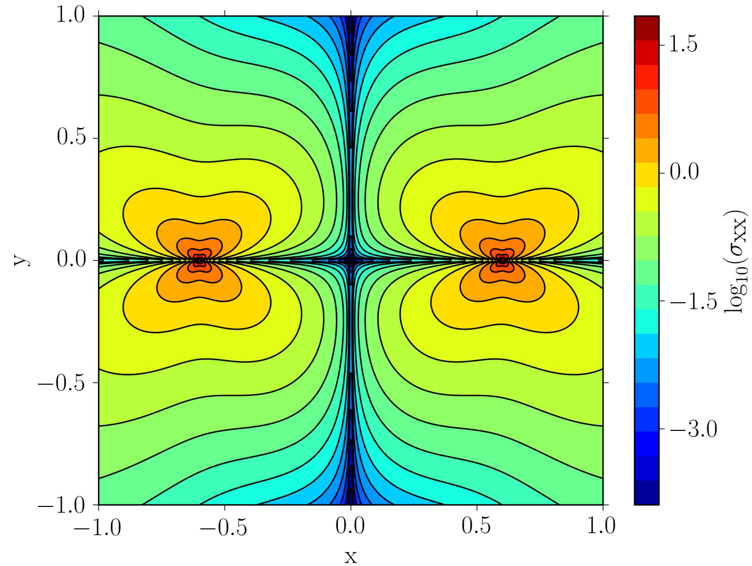

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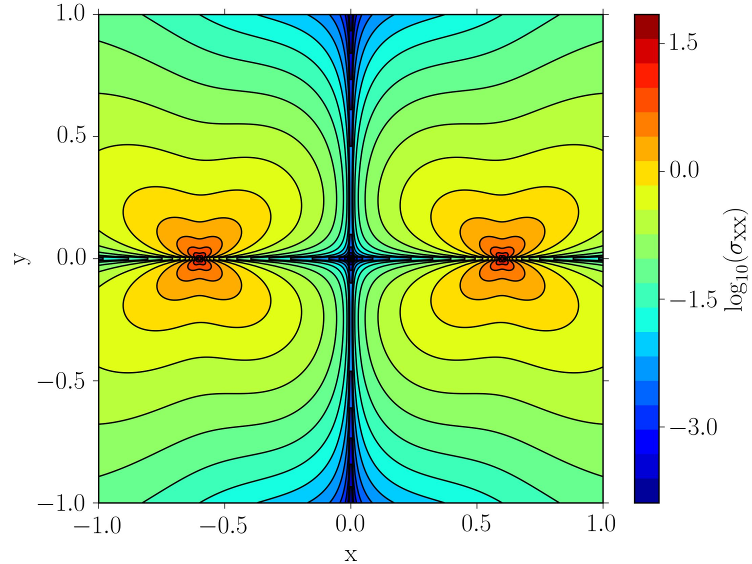
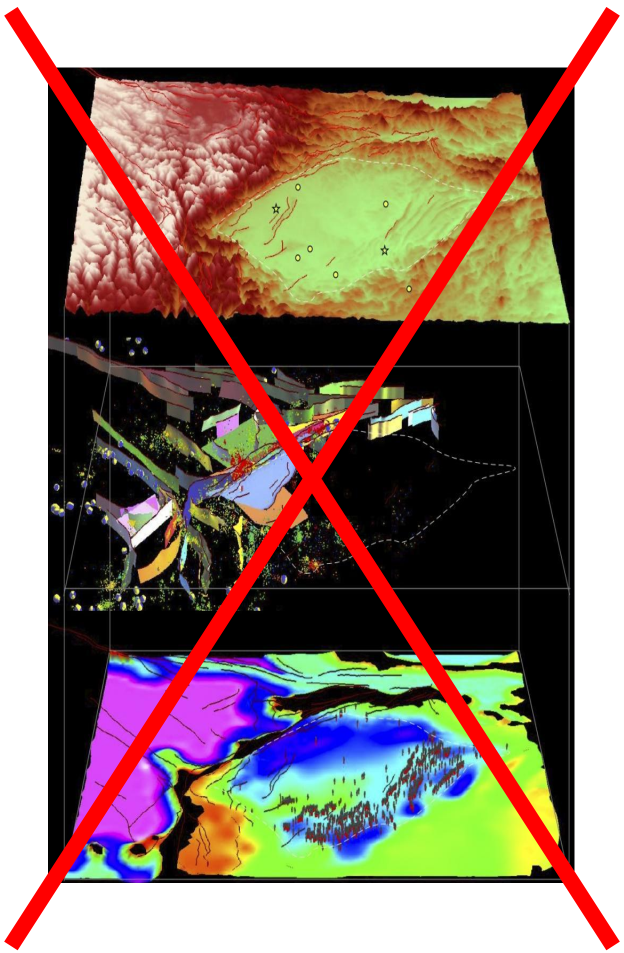


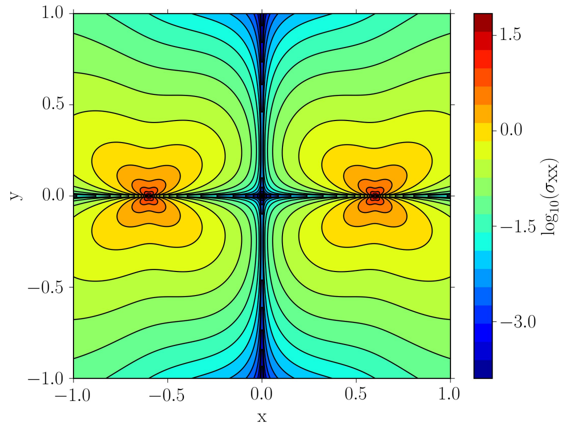


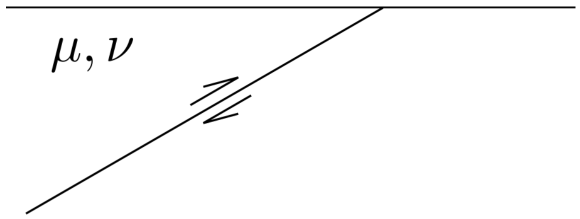
Simple models

Small models (**<50k** elements)

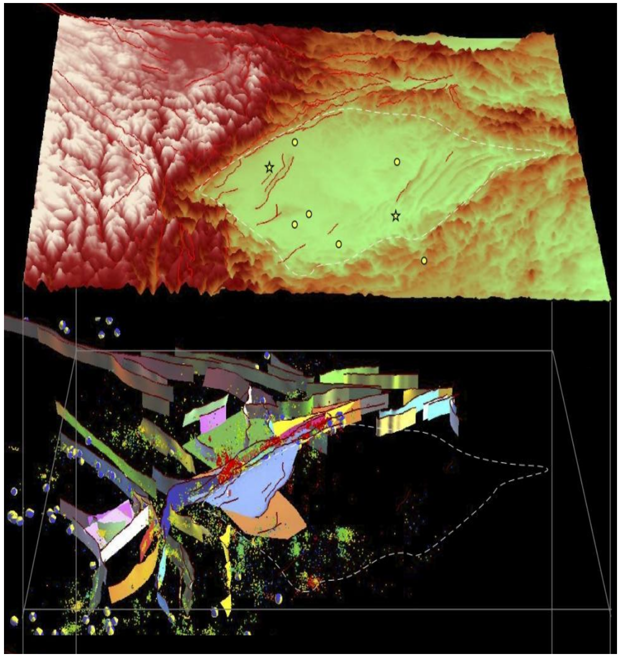
Current boundary element methods are also limited

What we usually do:
- Constant slip Okada dislocations produce unphysical singular stresses
- Geometrically simple models with <50,000 elements



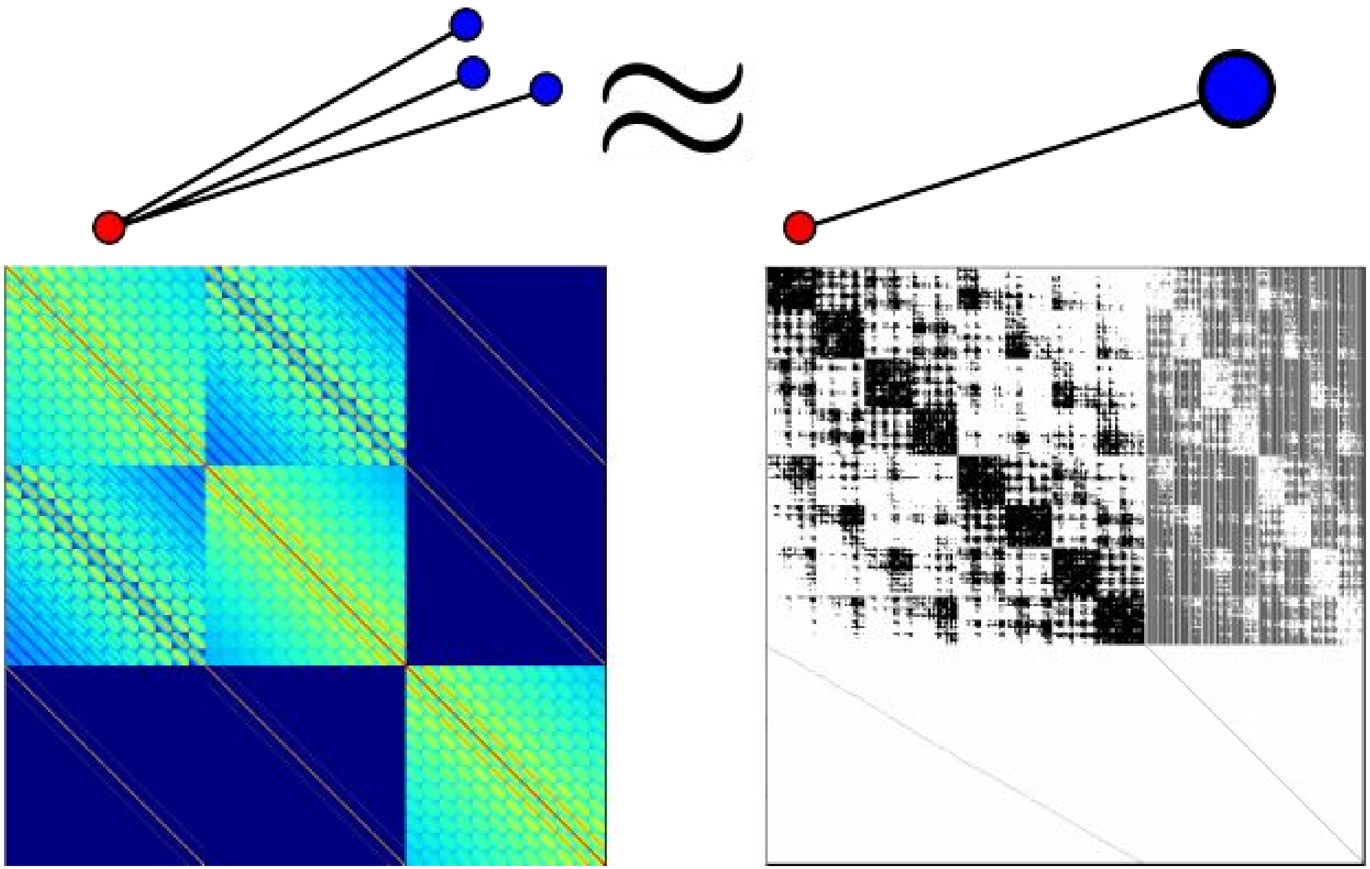


What we want to do:
- Non-singular stresses
- Large and geologically accurate models



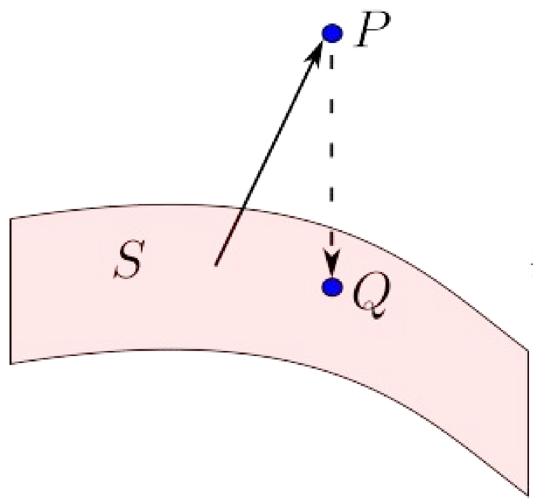
Far-field integrals are easy to compute, but numerous

→ Dense BEM matrices require O(N^2) entries
→ The Kernel Independent Fast Multipole Method can approximate BEM matrices with O(N) entries



Richardson Extrapolation is Accurate but Slow

A kernel independent approach for the near field integrals: **Richardson extrapolation**



$$\lim_{P \to Q} \int_S K(P,x) f(x) dx$$

$$\begin{aligned} I(x+h) &= I(x) + C_1 h \qquad &+O(h^2) \\ I(x+h/2) &= I(x) + C_1 h/2 \qquad &+O(h^2) \\ I(x+h) - 2I(x+h/2) &= I(x) \qquad &+O(h^2) \end{aligned}$$

General outlining:
-- BEM Methods -- I copied the AGU2016 slides, currently they're just images, so they're fuzzy, but I'll convert them later.

I want to think about the right way to modify these for the audience, since I think it's a bit different from the AGU audience. Also, the talk is less strictly about the methods and much more broad.

I think the BEM section should be feel like two parts:

-- Why?

-- How?

-- Should I show a quick confirmation that things work? Okada? Sphere to point out that we're not stuck in fault-only-world?

-- Idealized problems showing what can happen with topography

-- Wenchuan problem

Where to next?

- Use real models of the faults in Fialko 2005 (Bam, Hector Mine, Landers, Izmit)
- Spherical free surface?